A NOTE ON TRADE RESTRICTIONS AND RECIPROCAL DUMPING TRADE:
A MODEL OF AN INFINITELY REPEATED TRADE GAME*

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1. INTRODUCTION

It was shown in the models of Brander (1981), Brander and Krugman (1983), and so on that the reciprocal dumping trade (or intra-industry trade) of identical products is induced by the rivalry behavior of international duopolistic firms, given the segmented market. This is because the firm will choose the dumping trade strategy in the static one-shot game, no matter what the rival does. In other words, a reciprocal dumping trade equilibrium holds because of a prisoners' dilemma. Also, in this case the imposition of trade restrictions, i.e., tariffs and quotas, does not change the dumping trade strategy of the firm. But, Pinto (1986) proved that there can be a non- trade (or non -competitive) equilibrium if the international duopolistic competition is infinitely repeated[1]. That is, the firms will implicitly collude with each other in an infinitely repeated game if the discounted factor is sufficiently large. Then the firm will choose the non-exporting strategy in the long-run. Also, he concluded that the reciprocal dumping trade likely takes place if transfer costs (or tariffs) are negligible. However, opposed to his conclusion, it will be shown in our model that the more severe such trade restrictions as quotas, the more likely it is that the reciprocal dumping trade will occur.

The important aspect of our model is that the imposition of trade restrictions can reduce the threat of retaliation for the deviation from the implicit collusion. That is, if there are very restrictive quotas imposed by the government, then the firm will readily deviate from the implicit collusion, in which it will not export to the rival's market[2)]. On the contrary, with large quotas, the firm will not export to the rival's market in order to evade the rival's retaliation (i.e., the dumping trade) in the long-run.

In this paper we shall investigate how such trade restrictions as quotas can affect the strategies of the firms, when they play an infinitely repeated game in the international duopolistic market. We will show that the more severe the trade restrictions, the more likely it is that the reciprocal dumping trade will take place. This is because very restrictive quotas have the effect of reducing the threat of retaliation for the deviation from the implicit collusion, in which the firms will choose the non-exporting strategy.

In Section 2 we shall present the model and show that a reciprocal dumping trade equilibrium holds because of a prisoners' dilemma in the static one-shot game. In Section 3 we shall show the relationship between the equilibrium strategy of the firm and the amount of quotas in an infinitely repeated game. Finally, in Section 4 we will summarize our results and suggest some remaining problems of our model.

2. THE MODEL

2.1 SETTING

We assume that:

(1) there are two, i.e., the home and the foreign, countries $(i=H, F)$, where one monopolist $(j=1, 2)$ exists in each country and it produces identical products.

(2) there are no production costs, but positive transfer costs $(t>0)$[3].

(3) there are trade restrictions, i.e., quotas.

(4) there are such linearly inverse demand functions as

$P_i = A - Q_i,\ i=H, F,$

where $Q_i = \Sigma_j x_{ji},\ j=1, 2,\ x_{1H}, x_{2F} \geqq 0,\ 0 \leqq x_{2H}, x_{1F} \leqq \overline{x_{2H}}, \overline{x_{1F}}$. Note that Q_i is the total sale in country i, x_{ji} the sale of firm j for the market of country i, $\overline{x_{2H}}$ $(\overline{x_{1F}})$ the quotas for the export of firm 2 (1)

Therefore, taking into account the above assumptions, we derive the profit function of firm 1 as follows:

$\Pi_1 = P_H X_{1H} + (P_F - t) x_{1F},\ 0 \leqq x_{1F} \leqq \overline{x_{1F}}.$

Similarly, that of firm 2 can be derived.

2.2 ONE-SHOT GAME AND A PRISONERS' DILEMMA

Here we shall show the conditions wherein a reciprocal dumping trade equilibrium holds in the static one-shot game with quotas. We shall derive non-cooperative, cooperative, and deviative solutiuons, however, omitting the detailed calculation, we will present the derived equilibrium sales and profits in Table 1.

Table 1. Payoff Matrix

2 \ 1	M	D
M	$\Pi_1{}^M$ / $\Pi_2{}^M$	$\Pi_1{}^D$ / $\Pi_2{}'$
D	$\Pi_1{}'$ / $\Pi_2{}^D$	$\Pi_1{}^N$ / $\Pi_2{}^N$

(a) Non-Cooperative Cournot-Nash solution: (D, D)

$x_{1H}{}^N = (A - \overline{x_{2H}})/2$, $x_{1F}{}^N = \overline{x_{1F}} \leqq (A-2t)/3$,

$x_{2H}{}^N = \overline{x_{2H}} \leqq (A-2t$ $/3$, $X_{2F}{}^N = (A - \overline{x_{1F}})/2$.

$\Pi_1{}^N = (A - \overline{x_{2H}})^2/4 + \{(A-2t) - \overline{x_{1F}}\}\overline{x_{1F}}/2$, and

$\Pi_2{}^N = (A - \overline{x_{1F}})^2/4 + \{(A-2t) - \overline{x_{2H}}\}\overline{x_{2H}}/2$,

(b) Cooperative solution: (M, M)

$x_{1H}{}^M = A/2 = x_{2F}{}^M$, $x_{1F}{}^M = 0 = x_{2H}{}^M$.

$\Pi_1{}^M = A^2/4 = \Pi_2{}^M = A^2/4$.

(c) Deviation: (D, M) or (M, D)

$x_{1H}{}^D = A/2$, $x_{1F}{}^D = \overline{x_{1F}}$, for $0 \leqq \overline{x_{1F}} < (A-2t)/4$, and

$x_{1F}{}^D = (A-2t)/4$, for $(A-2t)/4 \leqq \overline{x_{1F}} < (A-2t)/3$.

$x_{2H}{}' = 0$, and $x_{2F}{}' = A/2$.

$\Pi_1{}^D = A^2/4 + \{A-2t) - 2\overline{x_{1F}}\}\overline{x_{1F}}K2$, for $0 \leqq \overline{x_{1F}} < (A-2t)/4$,and

$\Pi_2{}' = A(A - 2\overline{x_{1F}}/4$.

$\Pi_1{}^D = A^2/4 + (A-2t)^2/16$, for $(A-2t)/4 \leqq \overline{x_{1F}} < (A-2t)/3$, and

$\Pi_2{}' = A(A+2t)/8$.

Therefore, we have the payoff bimatrix (see Table 1). M (D) denotes the cooperative (non-cooperative) strategy of the firm. Note that the cooperative (non-cooperative) strategy stands for the non-exporting (the dumping trade). If both firms choose M, then the cooperative equilibrium holds. Then there is the no trade equilibrium, in which the firm will not export to its rival's market. But, if one of the firms will deviate from the cooperative equilibrium, it will export to its rival's market. Also, if both firms choose D, then the non-cooperative Cournot-Nash equilibrium holds. Then there is the reciprocal dumping trade equilibrium.

We shall show the conditions where a prisoners' dilemma holds. It can readily be proved that $\Pi_j^M < \Pi_j^D$ and $\Pi_j' < \Pi_j^N$, for $0 < \overline{x_{1F}}, \overline{x_{2H}} \leqq (A-2t)/3$, $j=1, 2$, where we assume $7A > 20t$[4]. Thus, we understand that D is the dominant strategy for both firms, and that the reciprocal dumping trade is an unique Nash equilibrium in the static one-shot game.

By the way, comparing profits in the cooperative solution with those in the non-cooperative solution, we can derive from such a relationship that

$$\Pi_1^M > \Pi_1^N \leftrightarrow \overline{x_{2H}}(2A - \overline{x_{2H}}) > 2\{(A-2t) - \overline{x_{1F}}\}\overline{x_{1F}}, \quad (1.1)$$

and

$$\Pi_2^M > \Pi_2^N \leftrightarrow \overline{x_{1F}}(2A - \overline{x_{1F}}) > 2\{(A-2t) - \overline{x_{2H}}\}\overline{x_{2H}}, \quad (1.2)$$

for $0 \leqq \overline{x_{1F}}, \overline{x_{2H}} \leqq (A-2t)/3$.

Thus, if the above conditions hold, a prisoners' dilemma occurs. Note that the above conditions always hold if quotas imposed are equal, i.e., $\overline{x_{1F}} = \overline{x_{2H}}$.

Therefore, we confirm that a reciprocal dumping trade equilibrium always holds in the static one-shot game with quotas, and that its equilibrium is not Pareto optimal for both firms, if (1.1) and (1.2) hold.

3. INFINITELY REPEATED GAME AND TRADE RESTRICTIONS

3.1 INFINITELY REPEATED GAME AND DISCOUNTED FACTOR

We assume that the firms will choose the grim triger strategy in an infinitely repeated game[5]. Then the implicit collusion between both firms holds, and there is the no trade equilibrium. The necessary and sufficient condition holding the implicit collusion is

$$\delta_i > \frac{\Pi_j^{D} - \Pi_j^{M}}{\Pi_j^{D} - \Pi_j^{N}},\ i=H,\ F,\ j=1,\ 2, \tag{2}$$

where $\delta_i = 1/(1+r_i)$ is the discounted factor of country i, and r_i the interest rate of country i. Also, if i=H(F), then j=1 (2). But, if such an unequal equation as

$$\delta_i \leqq \frac{\Pi_j^{D} - \Pi_j^{M}}{\Pi_j^{D} - \Pi_j^{N}},\ i=H,\ F,\ j=1,\ 2 \tag{3}$$

holds, then the firm will deviate from the implicit collusion, and it will export to its rival's market. Hence, the reciprocal dumping trade equilibrium holds in an infinitely repeated game. Note that the numerator of the right-hand side implies a gain from the deviation, and the denominator a loss from the retaliation. Also, note that the denominator is always positive if (1.1) and (1.2) hold. For simplicity, we assume that:

Assumption 1; the discounted factors of both countries are identical, i.e., $\delta_H = \delta_F = \delta$.

Therefore, taking into account equilibrium profits derived above, we can have the relationship as follows; As to firm 1,

$$\delta \gtreqless \frac{2\{(A-2t) - 2\,\overline{x_{1F}}\}\overline{x_{1F}}}{\overline{x_{2H}}(2A - \overline{x_{2H}}) - 2\,\overline{x_{1F}}^{2}} = \delta_H[\overline{x_{1F}},\ \overline{x_{2H}}], \tag{4.1}$$

for $0 \leqq \overline{x_{1F}} < (A-2t)/4,\ 0 \leqq \overline{x_{2H}} \leqq (A-2t)/3$,

and

$$\delta \gtreqless \frac{(A-2t)^2}{4\,\overline{x_{2H}}(2A - \overline{x_{2H}}) - 8\{(A-2t) - \overline{x_{1F}}\}\overline{x_{1F}} + (A-2t)^2}$$

$$= \delta_H' \left[\overline{x_{1F}}, \overline{x_{2H}}\right], \tag{5.1}$$

for $(A-2t)/4 \leqq \overline{x_{1F}} \leqq (A-2t)/3,\ 0 \leqq \overline{x_{2H}} \leqq (A-2t)/3.$

Similarly, as to firm 2,

$$\delta \gtreqless \frac{2\{(A-2t)-2\,\overline{x_{2H}}\}\overline{x_{2H}}}{\overline{x_{1F}}(2A-\overline{x_{1F}})-2\,\overline{x_{2H}}^2} = \delta_F\left[\overline{x_{1F}}, \overline{x_{2H}}\right], \tag{4.2}$$

for $0 \leqq \overline{x_{2H}} < (A-2t)/4,\ 0 \leqq \overline{x_{1F}} \leqq (A-2t)/3,$

and

$$\delta \gtreqless \frac{(A-2t)^2}{4\,\overline{x_{1F}}(2A-\overline{x_{1F}})-8\{(A-2t)-\overline{x_{2H}}\}\overline{x_{2H}}+(A-2t)^2}$$

$$= \delta_F' \left[\overline{x_{1F}}, \overline{x_{2H}}\right], \tag{5.2}$$

for $(A-2t)/4 \leqq \overline{x_{2H}} \leqq (A-2t)/3,\ 0 \leqq \overline{x_{1F}} \leqq (A-2t)/3.$

3. 2 TRADE RESTRICTIONS AND EQUILIBRIUM STRATEGY IN THE LONG-RUN

(a) LaissezFaire

At first, suppose that there are the non-trade restrictions, i.e., $\overline{x_{1F}} = \overline{x_{2H}} = (A-2t)/3$. Hence, (4.1)$\sim$(5.2) can be rewritten as follows:

$$\delta \gtreqless \frac{9(A-2t)}{13A+22t}. \tag{6}$$

Thus, if such an unequal equation as

$$\delta \leqq \frac{9(A-2t)}{13A+22t} \tag{7}$$

holds, the firms will deviate from the implicit collusion, and then a reciprocal dumping trade equilibrium holds. So, we assume that:

Assumption 2; the discounted factor of each country satisfies

$$\delta > \frac{9(A-2t)}{13A+22t}. \tag{8}$$

This assumption implies that a non-trade equilibrium holds because the firms will implicitly collude with each other under the laissez faire.

(b) Trade Restrictions

Here we shall discuss how quotas have effects on the strategies of firms in an infinitely repeated game. We shall first show it in the case of symmetric quotas, and secondly in the case of asymmetric quotas.

(i) Symmetric Quotas Case: $\overline{x_{1F}}=\overline{x_{2H}}=\overline{x}$

Taking into account the symmetry, (4.1) and (5.1) are equal to (4.2) and (5.2), respectively. Thus, we have

$$\delta \gtreqless \frac{2\{(A-2t)-2\overline{x}\}}{2A-3\overline{x}}=\delta[\overline{x}], \tag{9.1}$$

for $0\leqq\overline{x}<(A-2t)/4$,

and

$$\delta \gtreqless \frac{(A-2t)^2}{4\overline{x}(\overline{x}+4t)+(A-2t)^2}=\delta'[\overline{x}], \tag{9.2}$$

for $(A-2t)/4\leqq\overline{x}\leqq(A-2t)/3$.

Hence, from (9.1) and (9.2), we derive Lemma 1 as follows.

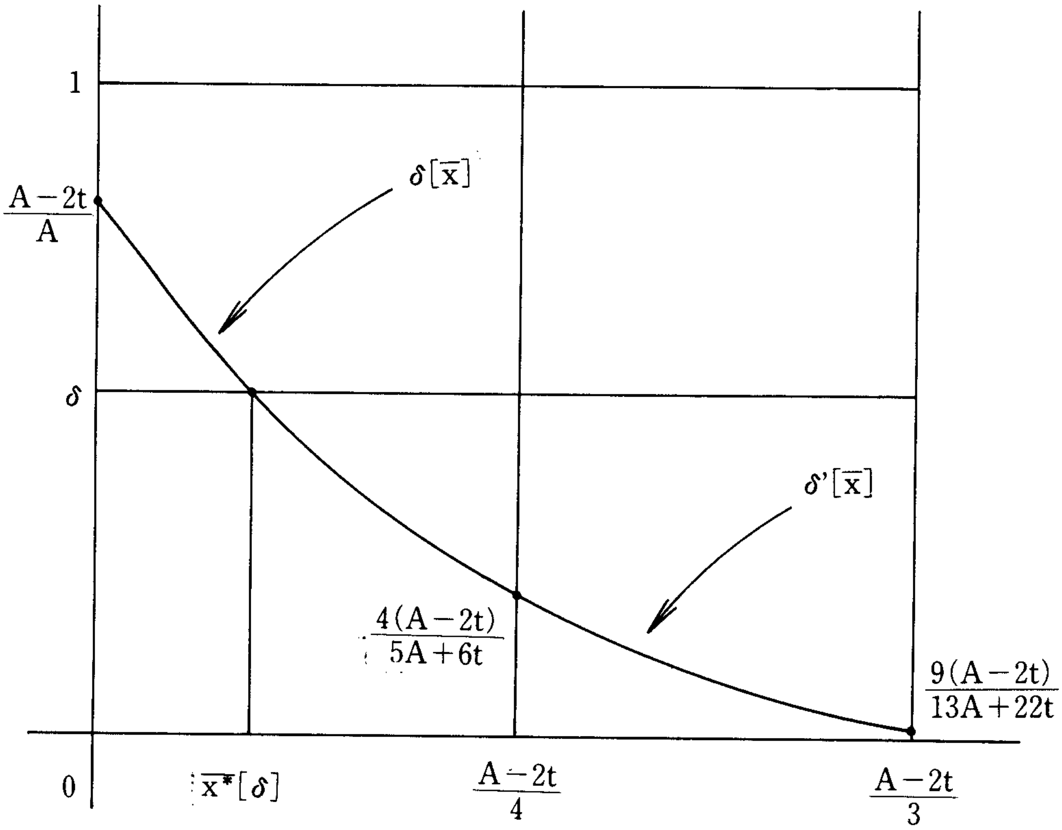
Lemma 1: If the discounted factor satisfies

$$\frac{A-2t}{A}>\delta>\frac{9(A-2t)}{13A+22t}, \tag{10}$$

such a boundary as $\overline{x}^*[\delta; A, t]$, for $0<\overline{x}^*[\delta; A, t]<(A-2t)/3$, exists. Then (a) if quotas ($\overline{x}$) exist within $0\leqq\overline{x}\leqq\overline{x}^*[\delta; A, t]$, the firms will deviate from the implicit collusion, and thus the reciprocal dumping trade equilibrium holds, and (b) if quotas ($\overline{x}$) exist within $\overline{x}^*[\delta; A, t]<(A-2t)/3$, the firms will implicitly collude, and thus a non-trade

Figure 1.1

$\frac{A-2t}{A} > \delta > \frac{4(A-2t)}{5A+6t}$

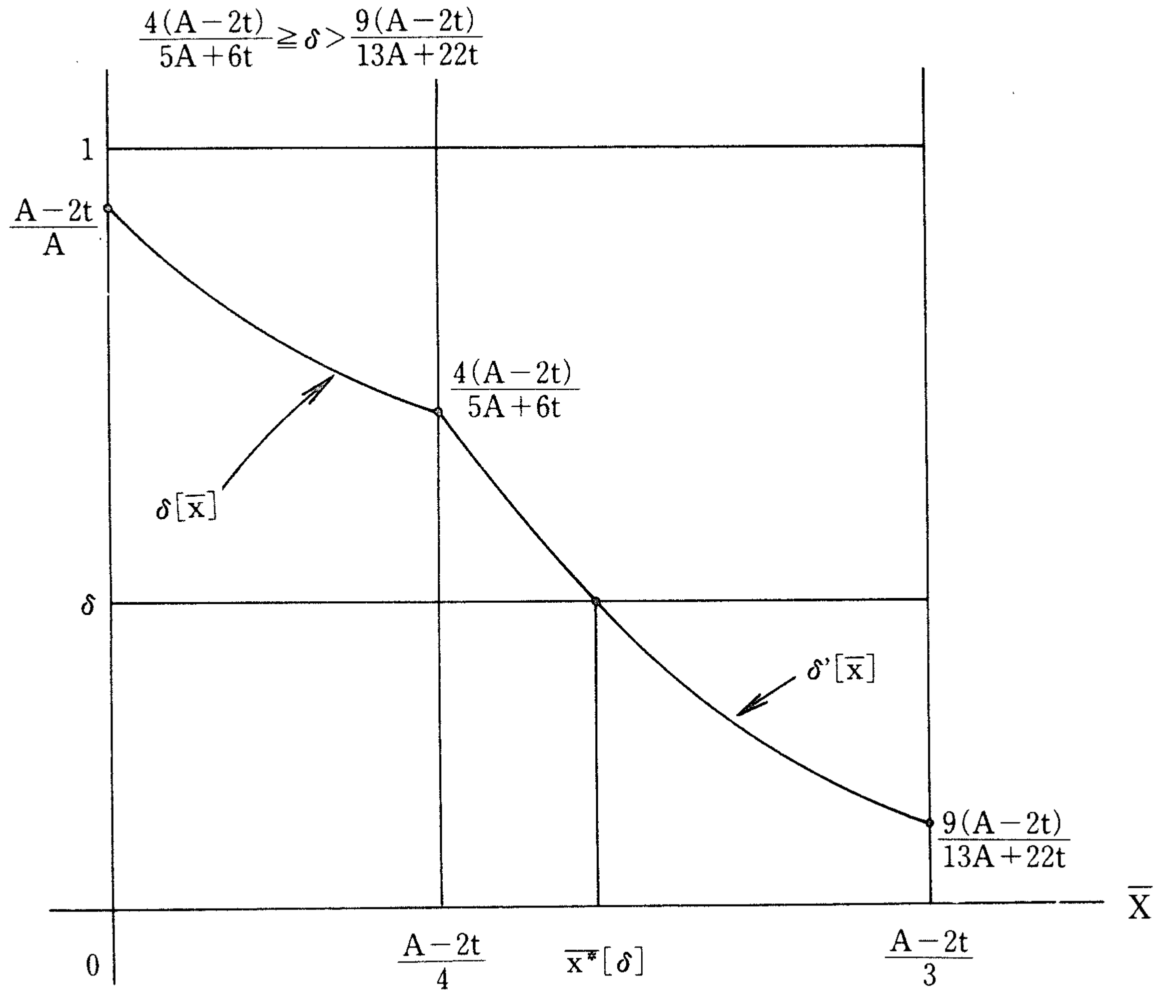


equilibrium holds. (See Figures 1.1 and 1.2.)

Proof. See Appendix 1.

This result means that the more restrictive the quotas, the more likely it is that the reciprocal dumping trade will take place[6]. That is, the firms will readily deviate from the implicit collusion since the strategy of the firm is affected by such effects that: the imposition of quotas clearly restrict the amount of the rival's export as well as that of the firm's export. In other words, very restrictive quotas decrease the threat of retaliation for the deviation from the implicit collusion as well as the firms' incentives to deviation. Thus, if the first effect can dominate the second, the reciprocal dumping trade is likely to take

Figure 1.2



place.

(ii)　Asymmetric Quotas Case: $\overline{x_{1F}} \neq \overline{x_{2H}}$

Finally, we will confirm the case of asymmetric quotas. In this case we can see that our assertion proved in Lemma 1 cannot basically be revised, and that there are two equilibria of the strategies besides the equilibria mentioned in Lemma 1. Here we will analyze the strategy of firm 1 since we can derive similar results as to that of firm 2.

First, (1.1) and (1.2) can be rewritten by

$$\overline{x_{2H}} > \Gamma[\overline{x_{1F}};\ A,\ t],\ \text{for}\ 0 \leqq \overline{x_{1F}} \leqq (A-2t)/3, \tag{11.1}$$

and

$\overline{x_{2H}} < \Lambda[\overline{x_{1F}};\ A,\ t]$, for $0 \leqq \overline{x_{1F}} \leqq (A-2t)/3$, (11.2)

Note that (11.1) and (11.2) are $\overline{x_{1F}} = \overline{x_{2H}}$ (i.e., 45^0) axis of symmetry, and hyperbola. Also, taking into account (4.1) or (5.1), we can see that firm 1 will deviate from the implicit collusion if it holds that

$\overline{x_{2H}} \leqq \Delta_1[\overline{x_{1F}};\ \delta,\ A,\ t]$, (12.1)

for $0 \leqq \overline{x_{1F}} < (A-2t)/4$, $0 \leqq \overline{x_{2H}} < (A-2t)/3$,

and

$\overline{x_{2H}} \leqq \Delta_1[\overline{x_{1F}};\ \delta,\ A,\ t]$, (13.1)

for $(A-2t)/4 \leqq \overline{x_{1F}} \leqq (A-2t)/3$, $0 \leqq \overline{x_{2H}} \leqq (A-2t)/3$.

Note that (12.1) and (13.1) are hyperbola.

If either (12.1) or (13.1) holds in the regions where (11.1) and (11.2) hold, then firm 1 will deviate from the implicit collusion. Therefore, taking into account (11.1), (11.2), (12.1), and (13.1), we can derive Proposition 1 and Corollary 1.

Proposition 1: Suppose that the discounted factor satisfies (10). Firm 1 will deviate from the implicit collusion, (i) if it holds that $\min\{\Delta_1[\overline{x_{1F}};\ \delta,\ A,\ t],\ \Lambda[\overline{x_{1F}}:\ A,\ t]\} \geqq \overline{x_{2H}} \geqq \Gamma[\overline{x_{1F}};\ A,\ t]$, for $0 < \overline{x_{1F}} < (A-2t)/4$, or (ii) if it holds that $\Delta_1'[\overline{x_{1F}};\ \delta,\ A,\ t] \geqq \overline{x_{2H}}, \geqq \Gamma[\overline{x_{1F}};\ A,\ t]$, for $(A-2t)/4 \leqq \overline{x_{1F}} \leqq (A-2t)/3$.

Proof. See Appendix 2.

Corollary 1: Suppose that the discounted factor satisfies (10). Firm 1 will not deviate from the implicit collusion, (i) if it holds that $\Lambda[\overline{x_{1F}};\ A,\ t] > \overline{x_{2H}} > \Delta_1[\overline{x_{1F}};\ \delta,\ A,\ t]$, for $0 < \overline{x_{1F}} < (A-2t)/4$, or (ii) if it holds that

$\Lambda[\overline{x_{1F}}; A, t] > \overline{x_{2H}} > \Delta_1'[\overline{x_{1F}}; \delta, A, t]$, for $(A-2t)/4 \leqq \overline{x_{1F}} \leqq (A-2t)/3$.

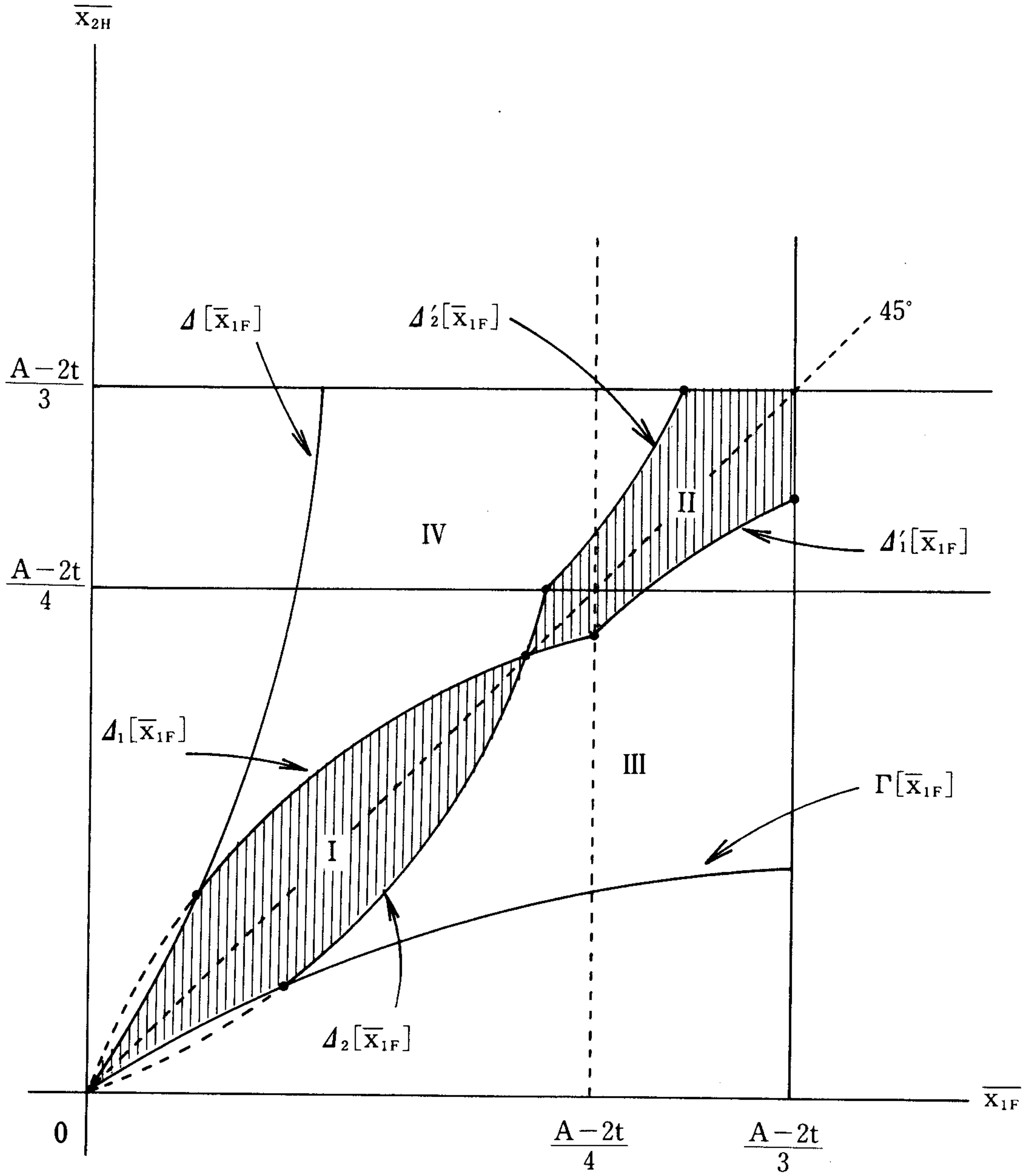
Proof. omit.

We can derive such similar equations as (12.1) and (13.1) as to firm 2, if $\overline{x_{1F}}$ and $\overline{x_{2H}}$ in (12.1) and (13.1) are changed, respectively. Denoting these equations (12.2) and (13.2), (12.2) and (13.2) are symmetrical to (12.1) and (13.1) at $\overline{x_{1F}} = \overline{x_{2H}}$ (i.e., 45^0) axis, respectively. Thus, we can also have the same results as Proposition 1 and Corollary 1 as to firm 2.

Therefore, we can draw the regions concerned with quotas imposed by both governments where equilibrium strategy of the firm holds in an infinitely repeated game (see Figures 2.1 and 2.2). In Region I both firms will deviate from the implicit collusion, so that a reciprocal dumping trade equilibrium holds. If the quotas imposed by both governments are in Region I, the firm will export the amount of quotas to the rival's market. In Region II both firms will implicitly collude, so that a non-trade equilibrium holds. If the quotas imposed by both governments are in Region II, the firm will not export to the rival's market, even if it can do. Comparing these regions, we can say that the smaller the quotas (i.e., the more severe the trade restrictions), the more likely the firm will deviate from the implicit collusion, so that it will export to the rival's market. Moreover, in Regions III and IV one of the firms will deviate from the implicit collusion, and the other will not. For instance, suppose that the quotas imposed by both governments are in Region III. Firm 1 will export the amount of quotas to country F, but firm 2 will not export, even if it can do. Thus, it is shown

Figure 2.1

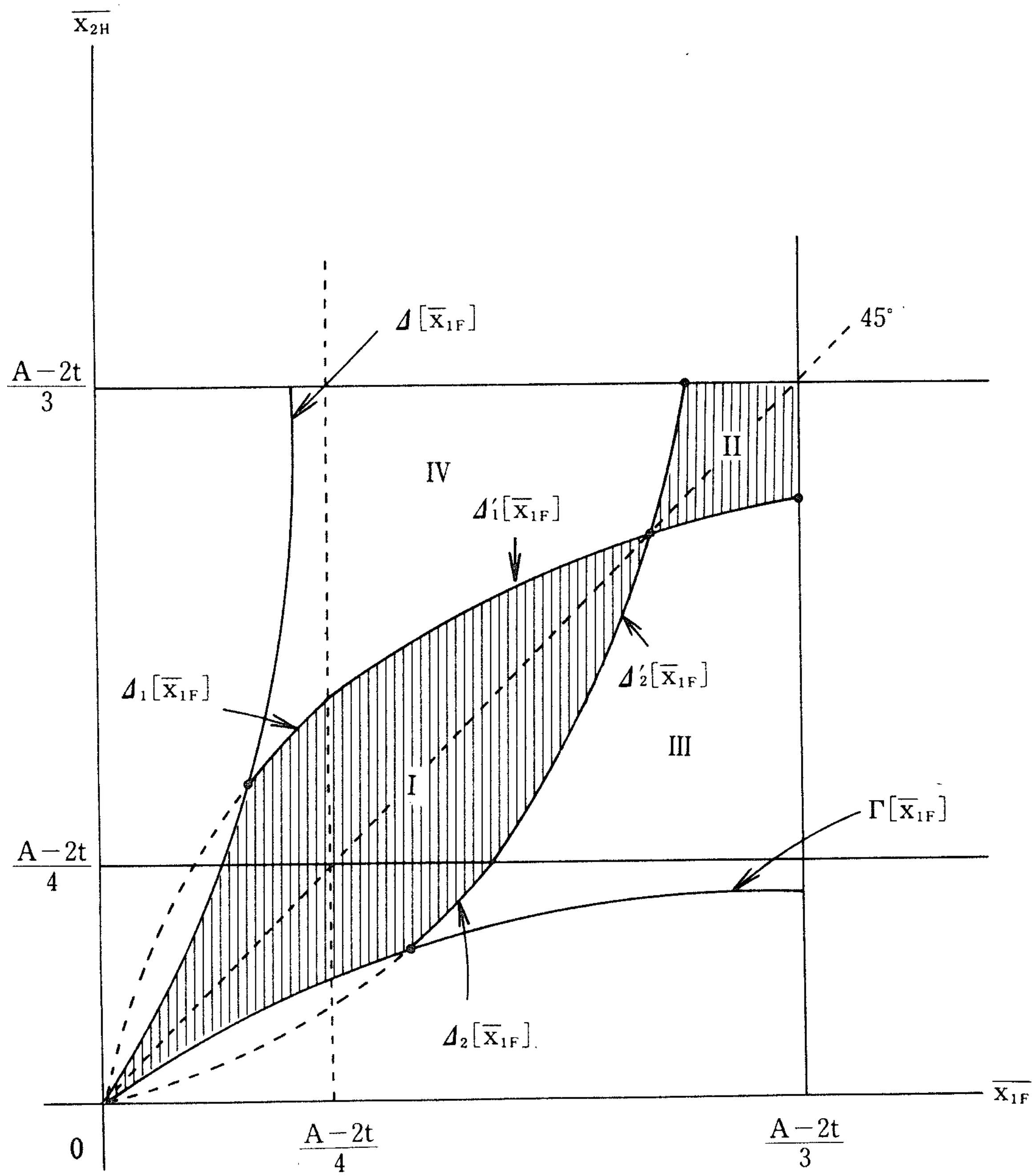
$\frac{A-2t}{A} > \delta > \frac{4(A-2t)}{5A+6t}$, and $\left(\frac{A-2t}{A}\right)^2 > \delta$. (＊)



(＊) Note that Δ_1 and Λ do not cross if it holds that $\{(A-2t)/A\}^2 < \delta$. Needless to say, taking into account the symmetry, we can similarly revise Δ_2 and Γ. Thus, even with the above case our main conclusion holds, although the space of Region I is just a little revised.

Figure 2.2

$$\frac{4(A-2t)}{5A+6t} \geqq \delta > \frac{9(A-2t)}{13A+22t}, \text{ and } \left(\frac{A-2t}{A}\right)^2 > \delta. \qquad (**)$$



(**) Note that Δ_1 and Λ do not cross if it holds that $(A\text{-}2t)/A\}^2 < \delta$. Needless to say, taking into account the symmetry, we can similarly revise Δ_2 and Γ. Thus, even with the above case our main conclusion holds, although the space of Region I is just a little revised.

in Regions III and IV that a 'one-way trade' equilibrium holds.

4. CONCLUDING REMARKS

Our main conclusion is that the trade restrictions can affect the strategies of the firms, when they infinitely compete in the international duopolistic market. That is, the tougher the trade restrictions, the more likely it is that the reciprocal dumping trade will take place. This is because, as mentioned above, the tougher the trade restrictions, the less the threat of retaliation. Hence, the firms will readily deviate from the implicit collusion, in which the no trade equilibrium holds. Thus, with such trade restrictions as quotas the firm is likely to export to the rival's market, although the imposition of quotas restricts the volume of the firm's export.

By the way, Pinto (1986) referred to the effects of transfer costs (or tariffs) on the strategies of the firms: the larger the transfer costs, the less likely it is that the reciprocal dumping trade will take place. In other words, when transfer costs are larger, the no trade equilibrium is likely to be supported by the threat of retaliation. See (6). The larger the transfer costs (or tariffs), the less the value of the right-hand side. Thus, the more severe such trade restrictions as transfer costs (or tariffs), the less likely it is that the reciprocal dumping trade will take place. This result is opposed to ours.

There are some remaining problems of our model: First, we shall reconsider the case of a finitely repeated game. As shown by Friedman

(1990), if we can make a model with multiple Nash equilibria in the static one-shot game, we will show that the implicit collusion can hold even with a finitely repeated game[7]. Thus, doing so, we can also analyze the same issue as that of the present paper in the case of a finitely repeated game. Secondly, the amount of quotas in our model were exogenously given, and thus the optimal trade policy was not explicitly treated[8]. If we will analyze the optimal trade policy, we shall consider it in the context of infinitely or finitely repeated policy games. Finally, our simple model can be extended to a more general one as to the following aspects: the domestic oligopoly, the asymmetric market scale and discounted factors of both countries, and so on.

Although it is very simple, our analysis may suggest that the imposition of trade restrictions, i.e., very restrictive quotas, can work as the export promotion in the international duopolistic market in the long-run. That is, with the severe trade restrictions, the strategy of the firm may move from the 'no trade' to the 'dumping trade' in an infinitely repeated game. Thus, it is possible for the domestic government to promote the export of the domestic firm as well as to protect the domestic firms against the export of the foreign firm by the severe trade restrictions in the long-run. Contrarily speaking, trade liberalization may not necessarily lead to a competitive market, but rather to an international collusion, i.e., a non-trade, in the long-run.

Appendix 1. Proof of Lemma 1.

First, as to (9.1), we have $\partial\delta[\bar{x}]/\partial\bar{x}<0$. Also, it is clear that

$$\lim_{\overline{x}\to 0} \delta[\overline{x}] = (A-2t)/A,$$

and

$$\lim_{\overline{x}\to (A-2t)/4} \delta[\overline{x}] = 4(A-2t)/(5A+6t) > 9(A-2t)/(13A+22t).$$

Second, as to (9.2), we have $\partial\delta[\overline{x}]/\partial\overline{x}<0$. Also, it is clear that

$$\delta'[\overline{x}=(A-2t)/4]=4(A-2t)/(5A+6t),$$

and

$$\delta'[\overline{x}=(A-2t)/3]=9(A-2t)/(13A+22t).$$

Thus, if (10) holds, there is such a boundary as $\overline{x}^*[\delta; A, t]$, for $0<\overline{x}^*[\delta; A, t]<(A-2t)/3$. Hence, (a) if $0\leqq\overline{x}\leqq\overline{x}^*[\delta; A, t]$, it holds that $\delta\leqq\delta[\overline{x}]$, for $(A-2t)/A>\delta> 4(A-2t)/(5A+6t)$, or it holds that $\delta\leqq\delta'[\overline{x}]$, for $4(A-2t)/(5A+6t)\geqq\delta>9(A-2t)/(13A+22t)$. Therefore, the firms will deviate from the implicit collusion. Also, (b) if $\overline{x}^*[\delta; A, t]<\overline{x}\leqq(A-2t)/3$ it holds that $\delta>\delta'[\overline{x}]$, for $(A-2t)/A>\delta>4(A-2t)/(5A+6t)$, or it holds that $\delta>\delta'[\overline{x}]$, for $4(A-2t)/(5A+6t)\geqq\delta>9(A-2t)/(13A+22t)$. Therefore, the firms will implicitly collude. Q.E.D.

Appendix 2. Proof of Proposition 1.

First, let us see the conditions, i.e., (1.1) and (1.2), where a prisoners' dilemma holds. From (1.1) and (1.2), we have

$$-2(A-\overline{x_{2H}})^2+4\left(\frac{A-2t}{2}-\overline{x_{1F}}\right)^2+(A^2+4At-4t^2)>0, \qquad (A.1)$$

and

$$-2(A-\overline{x_{1F}})^2+4\left(\frac{A-2t}{2}-\overline{x_{2H}}\right)^2+(A^2+4At-4t^2)>0, \qquad (A.2)$$

Note that (A.1) and (A.2) are a $\overline{x_{1F}}=\overline{x_{2H}}$ (i.e. 45^0) axis of symmetry, and hyperbola. Thus, we can derive the right-hand side of (11.1), i.e.,

$$\Gamma[\overline{x_{1F}};\ A,\ t]=A-\left(\frac{4\left(\frac{A-2t}{2}-\overline{x_{1F}}\right)^2+(A^2+4At-4t^2)}{2}\right)^{1/2} \tag{A.3}$$

where $0<\overline{x_{1F}}\leqq(A-2t)/3$. Also, we have the right-hand side of

$$\Lambda[\overline{x_{1F}};\ A,\ t]=\frac{A-2t}{2}-\left(\frac{2(A-\overline{x_{1F}})^2-(A^2+4At-4t^2)}{4}\right)^{1/2}, \tag{A.4}$$

where $0<\overline{x_{1F}}<A-\sqrt{(A^2+4At-4t^2)/2}<(A-2t)/3$. If $(A-2t)/3\geqq \overline{x_{1F}}\geqq A-\sqrt{(A^2+4At-4t^2)/2}$, then (1.2) always holds. Thus, from (A.3) and (A.4), we can readily show that:

$$\partial\Gamma[\overline{x_{1F}};\ t]/\partial\,\overline{x_{1F}}>0,\ \partial\Lambda[\overline{x_{1F}};\ A,\ t]/\partial\,\overline{x_{1F}}>0,$$

$$0<\lim_{\overline{x}_{1F}\to 0}\partial\Gamma[\overline{x_{1F}};\ A,\ t]/\partial\,\overline{x_{1F}}<1<\lim_{\overline{x}_{1F}\to 0}\partial\Lambda[\overline{x_{1F}};\ A,\ t]/\partial\,\overline{x_{1F}},$$

and

$$\Gamma[\overline{x_{1F}};\ A,\ t]<\Lambda[\overline{x_{1F}};\ A,\ t],\ \text{for}\ 0<\overline{x_{1F}}\leqq(A-2t)/3.$$

Therefore, the conditions, (1.1) and (1.2), hold, if and only if it holds that

$$\Gamma[\overline{x_{1F}};\ A,\ t]<\overline{x_{2H}}<\Lambda[\overline{x_{1F}};\ A,\ t],\ \text{for}\ 0<\overline{x_{1F}}\leqq(A-2t)/3.$$

Second, let us see (12.1) and (13.1). Taking into account Assumption 1 and (4.1), we can derive

$$-2\delta(2-\delta)(A-\overline{x_{2H}})^2+4(2-\delta)^2\left(\frac{A-2t}{2(2-\delta)}-\overline{x_{1F}}\right)^2-Z\gtreqless 0,$$

where $Z=(A-2t)^2-2\delta(2-\delta)A^2$. Hence, we have the right-hand side of (12.1), i.e.,

$$\Delta_1[\overline{x_{1F}};\ \delta,\ A,\ t]=A-\left(\frac{4(2-\delta)^2\left(\frac{A-2t}{2(2-\delta)}-\overline{x_{1F}}\right)^2-Z}{2\delta(2-\delta)}\right)^{1/2}, \tag{A.5}$$

where $0\leqq\overline{x_{1F}}<(A-2t)/4$. Note that $\partial\Delta_1[\overline{x_{1F}};\ \delta,\ A,\ t]/\partial\,\overline{x_{1F}}>0$, for

$0 \leqq \overline{x_{1F}} < (A-2t)/4$.

By a similar method as in (5.1), we can derive the right-hand side of (13.1), i.e.,

$$\Delta_1[\overline{x_{1F}};\ \delta,\ A,\ t] = A - \left(\frac{8\delta\left(\frac{A-2t}{2(2-\delta)} - \overline{x_{1F}}\right)^2 - Z'}{4} \delta \right)^{1/2}, \qquad (A.6)$$

where $Z' = 4\delta A^2 - (1+\delta)(A-2t)^2$, and $(A-2t)/4 \leqq \overline{x_{1F}} \leqq (A-2t)/3$. Also, note that $\partial \Delta_1'[\overline{x_{1F}};\ \delta,\ A,\ t]/\partial\, \overline{x_{1F}} > 0$, for $(A-2t)/4 \leqq \overline{x_{1F}} \leqq (A-2t)/3$.

Third, we can show the relationship of (A.3), (A.4), (A.5) and (A.6) as follows: From (A.3), (A.5), and (A.6), it holds that

$\Gamma[\cdot] < \Delta_1[\cdot]$, for $0 < (A-2t)/4$,

and

$\Gamma[\cdot] < \Delta_1'[\cdot]$, for $(A-2t)/4 \leqq \overline{x_{1F}} \leqq (A-2t)/3$.

From (A.4) and (A.5), it holds that

$$\lim_{\overline{x_{1F}} \to 0} \partial \Delta_1[\cdot]/\partial\, \overline{x_{1F}} \gtreqqless \lim_{\overline{x_{1F}} \to 0} \partial \Lambda[\cdot]/\partial\, \overline{x_{1F}} \leftrightarrow \{(A-2t)/A\}^2 \gtreqqless \delta.$$

where (A.4) is equal to (A.5) at $\overline{x_{1F}} = \overline{x_{1F}}^{\#}$. Also, as to (A.4) and (A.6), it holds that

$\Lambda[\cdot] > \Delta_1'[\cdot]$, for $(A-2t)/4 \leqq \overline{x_{1F}} \leqq (A-2t)/3$.

Now, taking into account the above discussion about (A.3), (A.4), (A.5) and (A.6), we will prove Proposition 1.

First, as to (i), if and only if it holds that

$\min\{\Delta_1[\overline{x_{1F}};\ \delta,\ A,\ t],\ \Lambda[\overline{x_{1F}};\ A,\ t]\} \geqq \overline{x_{2H}} \geqq \Gamma[\overline{x_{1F}};\ A,\ t]$

(11.1) holds for $0 \leqq \overline{x_{1F}} < (A-2t)/4$. Secondly, as to (ii), if and only if it holds that

$\Delta_1'[\overline{x_{1F}};\ \delta,\ A,\ t] \geqq \overline{x_{2H}} \geqq \Gamma[\overline{x_{1F}};\ A,\ t]$

(11.2) holds for $(A-2t)/4 \leqq \overline{x_{1F}} \leqq (A-2t)/3$.

Note that when (10) holds, taking into account Lemma 1, we can derive that: (A.5) crosses 45^0, i.e., $\overline{x_{2H}}=\overline{x_{1F}}$, from upwards at $2\{(1-\delta)A-2t\}/(4-3\delta)$, when $(A-2t)/A>\delta>4(A-2t)/(5A+6t)$ and (A.6) crosses 45^0 from upwards at $R(A-2t)-2t$, $R=\sqrt{(1-\delta)\delta}$, when $4(A-2t)/(5A+6t) \geqq \delta > 9(A-2t)/(13A+22t)$. Thus, we can have Figure 2.1, and 2.2. Q.E.D.

FOOTNOTE

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1) Fung (1991) proves in the case of differentiated products that there is the implicit collusion in which a reciprocal dumping trade equilibrium holds.

2) This idea is based on a 'topsy-turvey' principle. There are some models using this idea, for example, Davidson (1984), Krishna (1989), Rotenberg and Saloner (1989), and Syripoulos (1992). Especially, the last two papers are very related to ours. They discuss how such trade

policies as tariffs and quotas can affect the stability of the cartel in the domestic industry.

3) If transfer costs are zero, there are such two equilibria at the implicit collusion that: each firm only sells for its own market, or each firm sells for its rival's market and not for its own market.

4) If $7A < 20t$, then there are two Nash equilibria in the pure strategy and one equilibrium in the mixed strategy.

5) On the grim trigger strategy in repeated games see Friedman (1990).

6) Let us see the effects of parameters, market scale, A, and transfer costs, t, on the boundary $\overline{x}^*[\delta; A, t]$. We can derive that $\partial \overline{x}^*[\delta; A, t]/\partial A > 0$, and $\partial \overline{x}^*[\delta; A, t] \partial t < 0$. Thus, the larger the market scale, or the less the transfer costs, the more readily the firms will deviate from the implicit collusion.

7) For example, see Fraysse and Moreaux (1985), and Harrington (1987). Assuming that the firms must pay the fixed costs at every period, and will take the discriminating trigger strategy, they discuss the possibility of holding the implicit collusion in the context of the oligopoly game under a finite horizon.

8) See Davidson (1984) and Syripoulos (1992). They analyzed the optimal tariffs and quotas of the importing country in the context that the firms play an infinitely repeated game in the international oligopolistic market.

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