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The Effectiveness of Non-Linear Crack Mechanics Based on Elastic-Plastic Behavior near a Crack Tip

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き裂先端近傍の弾塑性挙動に基づく非線形き裂力学の有効性

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Abstract: The fracture of machines and structures is brought by the growth of a crack in most cases. Moreover the fracture occurs in some cases after producing large plastic deformation near the crack tip. Therefore the measure of mechanical severity of a cracked body in such a case is necessary to assure the occurrence of same phenomena in a cracked specimen and a real cracked object under large scale yielding. Recently, the non-linear crack mechanics was proposed by one of the authors. In this concept, the plastic strain at a crack tip obtained by the finite element method (FEM) is used as the measure of mechanical severity of a cracked body. In this paper, the effectiveness of the non-linear crack mechanics is discussed through the elastic-plastic behavior near a crack tip obtained by the FEM analysis under small scale yielding and large scale yielding.

Keywords: Plasticity, Crack, Non-Linear Crack Mechanics, *J*-Integral, Finite Element Method

Introduction

In general the failure or fracture of machines and structures is brought by the growth of a crack in most cases. As well known, the linear fracture mechanics [1] can play an important role to predict fracture or yield near a crack tip under small scale yielding. That is, the stress intensity factor is an effective measure of severity controlling failure or fracture in a cracked body under small scale yielding. On the other hand, the fracture of a real object occurs in some cases after producing large plastic deformation near the crack tip. In this case, it is necessary to know the measure of mechanical severity of a cracked body under large scale yielding.

The *J*-integral has been used widely in treating the crack problems under large scale yielding [2]. It is assumed that the *J*-integral value is independent of integral path in most cases. However, the dependence of integral path has been indicated by some researchers [3, 4]. On the other hand, the non-linear crack mechanics [5] has been proposed by one of the authors. In the non-linear crack mechanics, the plastic strain at a crack tip obtained by FEM is used as the measure of severity of a cracked body. The effectiveness of the non-linear crack mechanics has been confirmed through a few results of FEM analysis and a comparison with the experimental data until now [4-6].

In this paper, the effectiveness of the non-linear crack mechanics is discussed through the elastic-plastic behavior near a crack tip obtained by the FEM analyses in the cases of the cracked plate specimens under small scale yielding and large scale yielding subjected to tension. Moreover the characteristics of *J*-integral and the dependence on integral path in *J*-integral are examined by the elastic-plastic FEM analysis based on the concept of the non-linear crack mechanics.

Treated Problems and Conditions for Elastic-Plastic FEM Analysis

Figure 1 shows the dimensions of the plate specimens with a center crack or a pair of edge cracks treated in the present analyses. The FEM mesh pattern near the crack tips is shown in Fig. 1. The half widths w of the specimens are 10 mm and the half crack lengths a are 1 mm, 4 mm and 8 mm. In order to compensate the error due to the difference in FEM mesh patterns, the same mesh pattern shown in Fig.1 is used near the crack tips in all specimens. The elements near the crack tips are

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made fine systematically as shown in Fig. 1. The minimum element size at the crack tips is 1/243mm.

The relation between the true stress and the equivalent plastic strain used in the present elastic-plastic analyses is shown in Fig. 2. The elastic-plastic analyses were carried out by using Mises yield criterion and the incremental theory of plasticity. The FEM software Marc was used in the present elastic-plastic analyses.

In this study, as a measure of mechanical severity assuring the occurrence of the same phenomena, the value of plastic strain at a crack tip obtained by FEM $\varepsilon^p_{y0,FEM}$ is used as shown in Fig. 3. $\varepsilon^p_{y0,FEM}$ is the value of *y*-directional plastic strain at a crack tip, where *y* means the tensile direction.

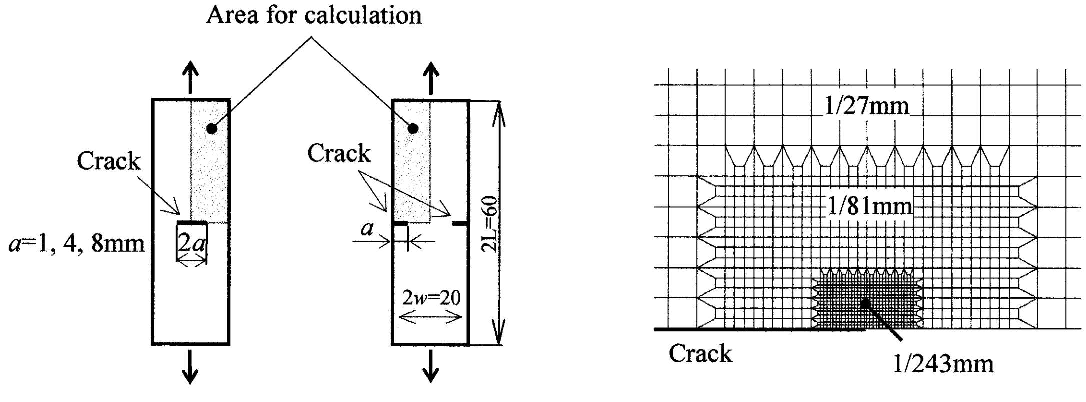
Discussion Based on Elastic-Plastic FEM Analysis

Figure 4 shows the nominal stress - nominal strain curves of the center crack specimens and edge crack specimens shown in Fig. 1. The nominal stress is defined as the stress of the specimen which is determined by neglecting the existence of a crack. The marks (△, ◇, ○, □) in these figures indicate the points at $\varepsilon^p_{y0,FEM}$=20%, 50%, 100% and 150%, respectively. These points also correspond to the plastic strain values at the crack tips in Figs. 5 and 6.

Non-Linear Crack Mechanics. In the following, the effectiveness of the non-linear crack mechanics is explained through the elastic-plastic behavior near a crack tip under small scale yielding and large scale yielding.

Under Small Scale Yielding. Figure 5 shows the crack opening shapes and the plastic strain distributions near the crack tips of the center crack specimens and edge crack specimens in the case of $\varepsilon^p_{y0,FEM}$=20%. These cases can be considered to be under small scale yielding, because the plastic zone sizes are very small.

When the plastic strain values at the crack tips $\varepsilon^p_{y0,FEM}$ are the same, the plastic strain distributions near the crack tips are almost the same, in spite of different crack length and different crack type as seen from Fig. 5. Therefore, $\varepsilon^p_{y0,FEM}$ can be considered as an effective parameter for assuring the occurrence of the same



(a) Center crack (b) Edge crack FEM mesh pattern near the crack tip

Fig. 1 Plate specimens with a center crack or a pair of edge cracks

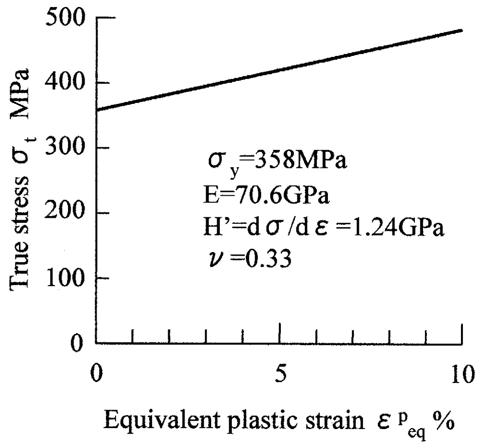


Fig. 2 True stress-equivalent plastic strain diagram

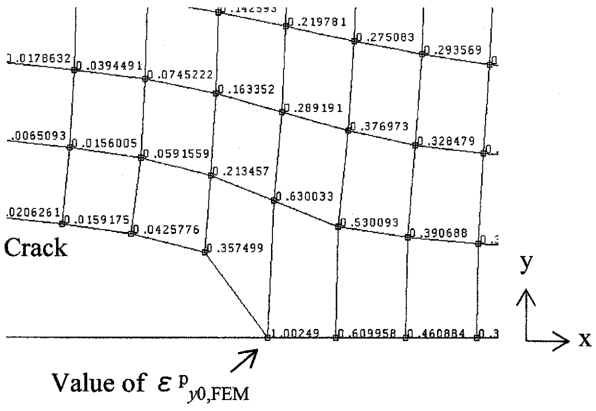
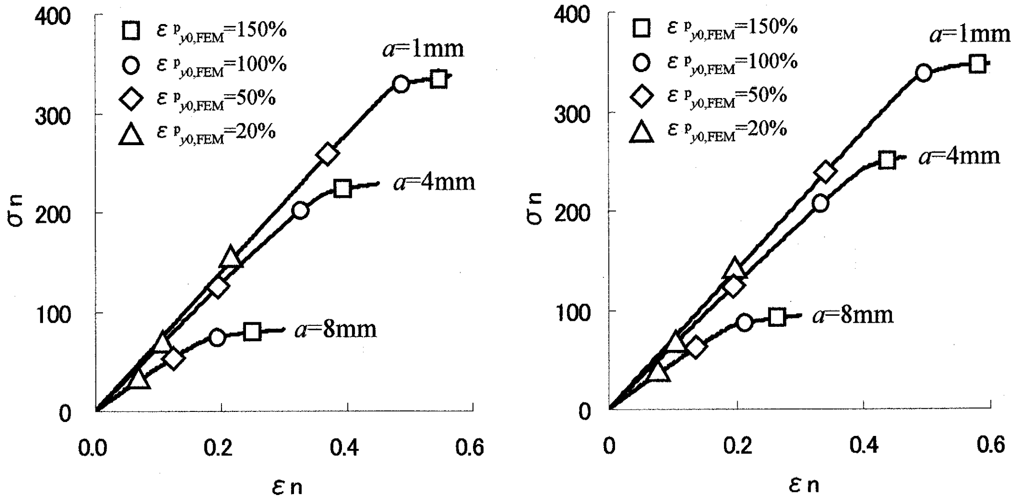


Fig. 3 Value of plastic strain at a crack tip



(a) Center crack specimen (b) Edge crack specimen

Fig. 4 Nominal stress–nominal strain curves

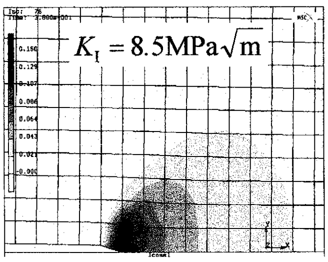
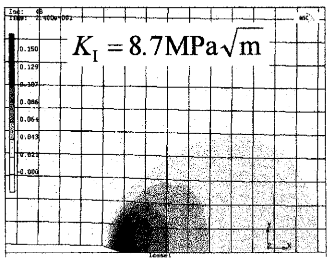
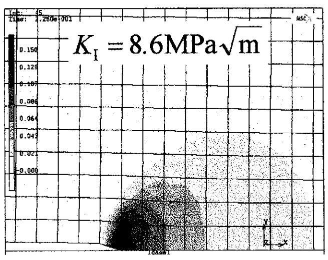
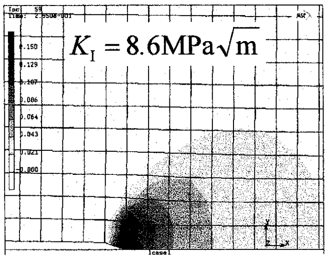
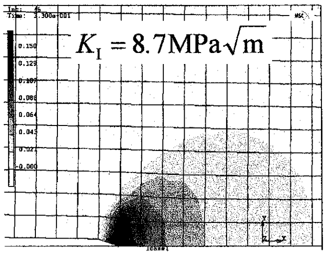
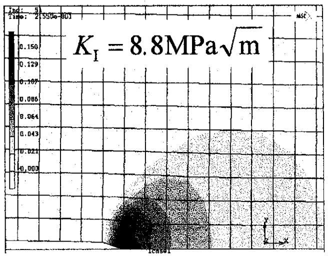
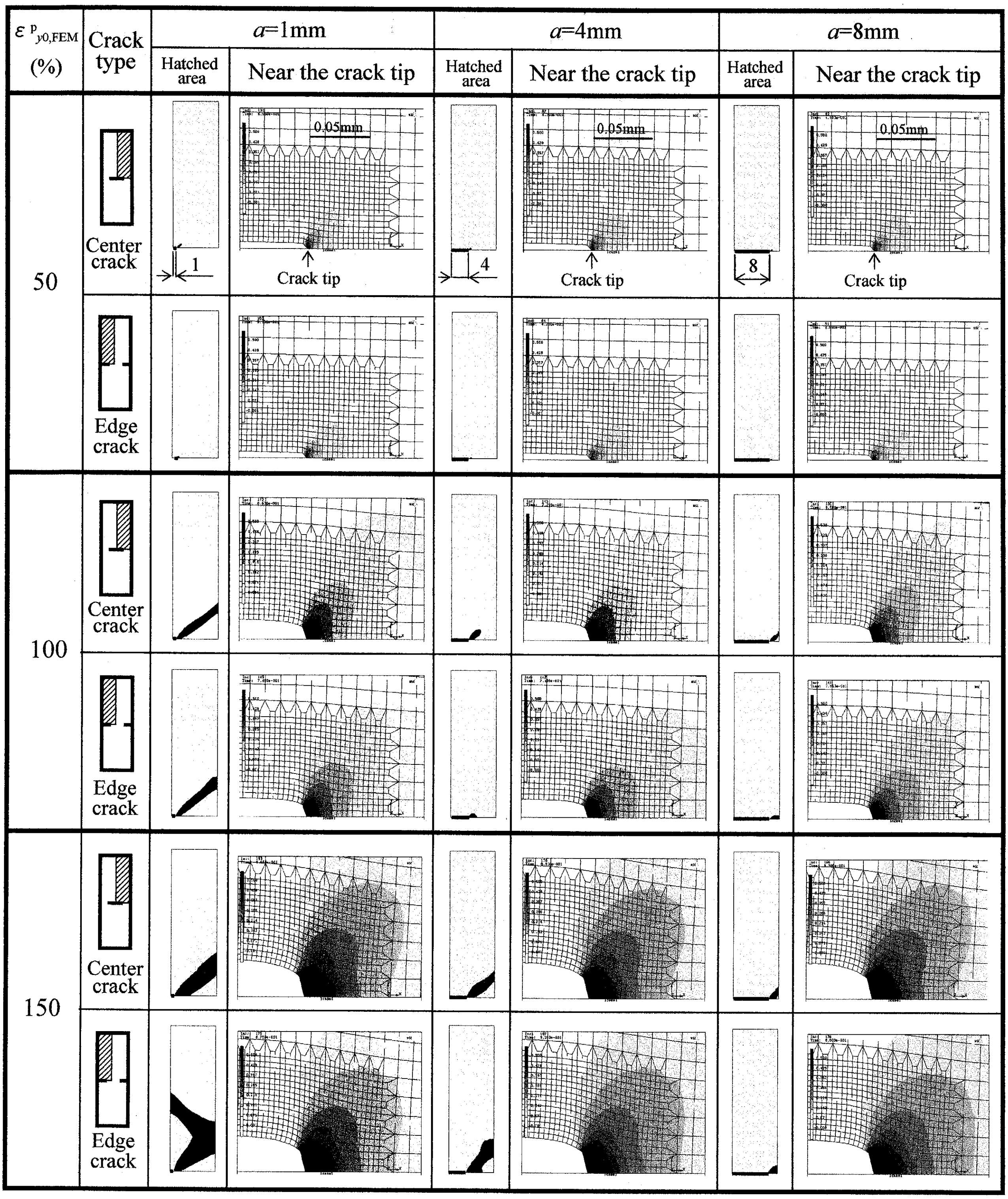
Crack type	$\varepsilon^p_{y0,\mathrm{FEM}}$=20%		
	a=1mm	a=4mm	a=8mm
Center crack	 $K_\mathrm{I} = 8.5\mathrm{MPa}\sqrt{\mathrm{m}}$	 $K_\mathrm{I} = 8.7\mathrm{MPa}\sqrt{\mathrm{m}}$	 $K_\mathrm{I} = 8.6\mathrm{MPa}\sqrt{\mathrm{m}}$
Edge crack	 $K_\mathrm{I} = 8.6\mathrm{MPa}\sqrt{\mathrm{m}}$	 $K_\mathrm{I} = 8.7\mathrm{MPa}\sqrt{\mathrm{m}}$	 $K_\mathrm{I} = 8.8\mathrm{MPa}\sqrt{\mathrm{m}}$

Fig. 5 Plastic strain distribution under small scale yielding

phenomena under small scale yielding, and therefore the non-linear crack mechanics is effective even under small scale yielding.

Under Large Scale Yielding. Figure 6 shows the plastic strain distributions and the crack opening shapes of the center crack specimens and edge crack specimens in the cases of $\varepsilon^p_{y0,\mathrm{FEM}}$=50%, 100% and 150%. The hatched area in the specimens means the analyzed area. The black zone in the analyzed areas means a plastic zone. Figure 7 shows the comparison of the crack opening shapes near the crack tips of the center crack specimens and edge crack specimens in the cases of $\varepsilon^p_{y0,\mathrm{FEM}}$ = 20%, 50%, 100% and 150%.

When the values of $\varepsilon^p_{y0,\mathrm{FEM}}$ in these cases are the same, the plastic strain distributions and the crack opening shapes near the crack tips are almost the same, in spite of the different crack length and different crack type, as seen from Figs. 6 and 7. Especially in the cases



▬ in analyzed area : Plastic zone

Fig. 6 Plastic strain distributions and crack opening shapes of the specimens shown in Fig.1

of $\varepsilon^{p}_{y0,FEM}$=100%, although the specimens of a=1 mm and 8 mm are general yield and the specimens of a=4 mm are local yield, the plastic strain distributions and crack opening shapes are almost the same independent of a, as seen from Fig. 6. Therefore, $\varepsilon^{p}_{y0,FEM}$ is an effective parameter in elastic-plastic crack problems. That is, this parameter is an effective measure of mechanical severity assuring the occurrence of the same

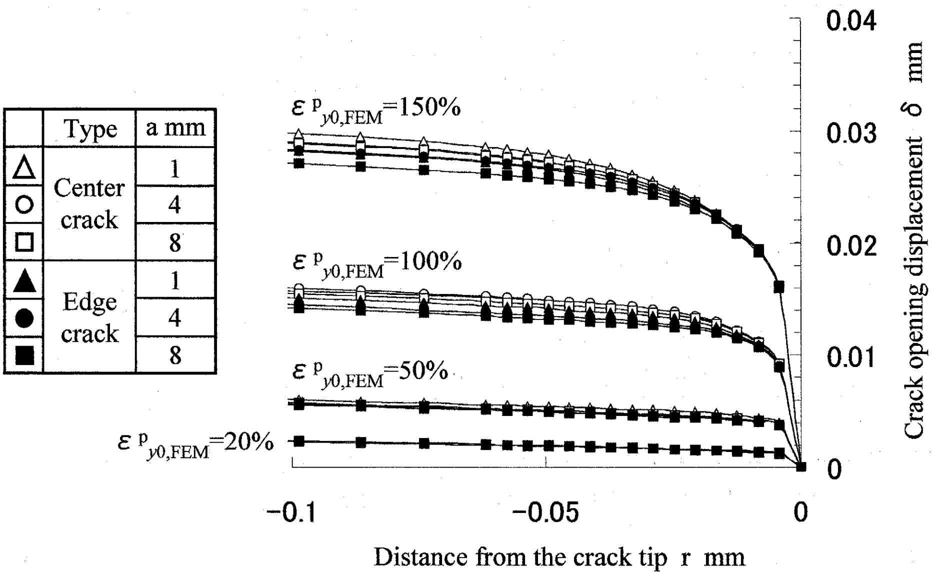


Fig. 7 Crack opening shapes near the crack tips

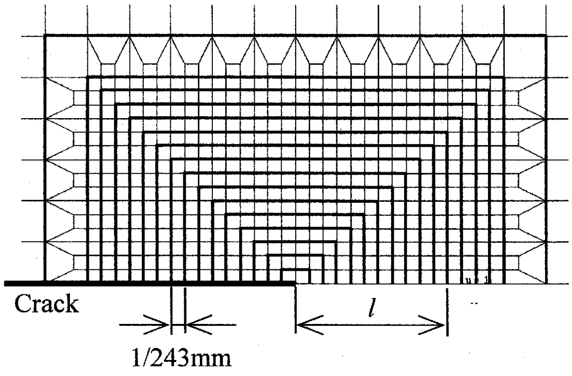
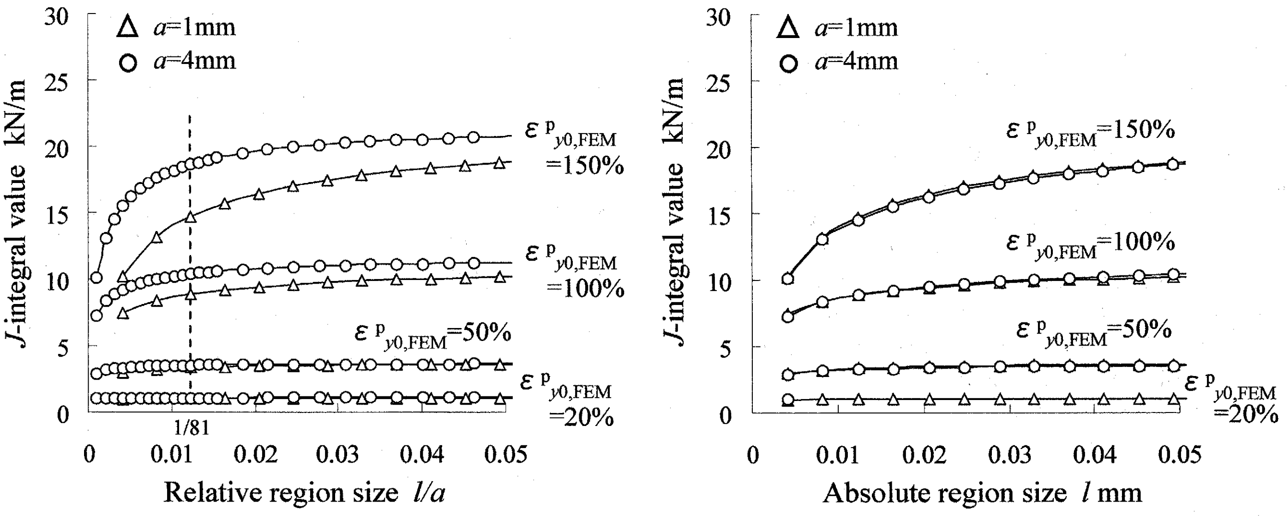


Fig. 8 Region for obtaining J-integral value



(a) Relative region size

(b) Absolute region size

Fig. 9 J-integral value obtained by the rigid region method (Center crack specimen)

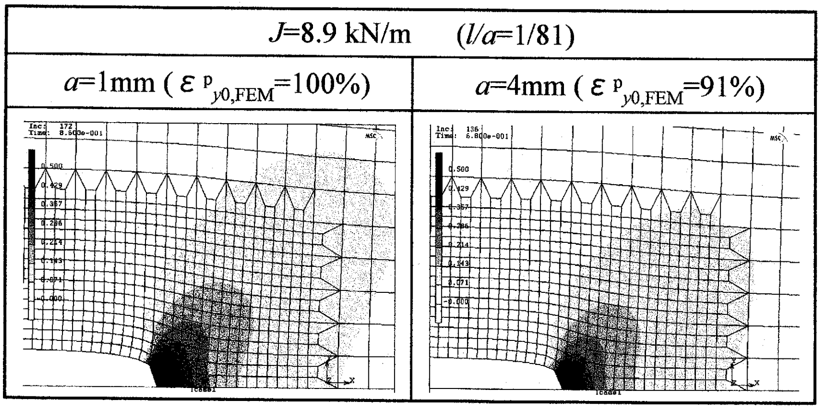


Fig. 10 Plastic strain distribution and crack opening shape under the condition of *J*-integral value=constant (Center crack specimen)

phenomena in the range from small scale yielding to large scale yielding.

***J*-Integral Method.** The values of *J*-integral were obtained by using the rigid region method [7], which is included in the used FEM software Marc. Figure 8 shows the regions for obtaining *J*-integral values. The symbol *l* in this figure means the size of the region for obtaining *J*-integral values. Figure 9 shows the *J*-integral values of the center crack specimens under the condition of the same $\varepsilon^p_{y0,\mathrm{FEM}}$ ($\varepsilon^p_{y0,\mathrm{FEM}}$=20%, 50%, 100% and 150%). The crack lengths of specimens treated in Fig. 9 are 1mm and 4 mm. Figure 9 (a) shows the relation between the *J*-integral value and the relative region size *l*/*a*. Figure 9 (b) shows the relation between the *J*-integral value and the absolute region size *l*. Figure 10 shows the plastic strain distributions and the crack opening shapes under the same *J*-integral value=8.9 kN/m. These *J*-integral values were obtained under the condition of the same relative region size *l*/*a* = 1/81.

In Fig.9 (a), the *J*-integral-*l*/*a* relations of *a*=1mm and *a*=4mm do not coincide under large plastic deformation ($\varepsilon^p_{y0,\mathrm{FEM}}$ = 100% and 150%). As seen from Fig. 10, the plastic strain distributions and the crack opening shapes of the two specimens are different in spite of the same *J*-integral value. However, if we use the same region size for obtaining the *J*-integral value, the curves of *a*=1 mm and *a*=4 mm almost coincide (Fig. 9 (b)), although the *J*-integral values are path dependent. This is due to the fact that the elastic-plastic strain distributions near the crack tips are almost the same in spite of the different crack length, if the values of $\varepsilon^p_{y0,\mathrm{FEM}}$ are the same.

Conclusion

The non-linear crack mechanics was applied to the center crack specimens and edge crack specimens under small scale yielding and large scale yielding in order to investigate the effectiveness of this method through the elastic-plastic behavior near a crack tip. As the results, when the values of the plastic strain at the crack tips obtained by FEM are the same, the elastic-plastic strain fields near the crack tips are almost the same in spite of the different crack length and different crack type. Then the same elastic-plastic strain fields should result in the same phenomena. This means that the non-linear crack mechanics is an effective concept which assures the occurrence of same phenomena in a cracked specimen and a real cracked object in the range from small scale yielding to large scale yielding.

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