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Spatial Evolutionary Prisoner's Dilemma Game by Diffusively Traveling Individuals

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1 Introduction

The origin of cooperation observed in a group consisted of egoistical individuals is an interesting subject in biology and human sciences. In order to make up a cooperative society, is any powerful force necessary? Or is cooperation fostered spontaneously? The evolutionary game theory has been introduced as an approach to such problems [1].

In the early evolutionary game theory, the system concerned has been supposed to be homogeneous. Its main interest was to find an evolutionary stable strategy that can be predominant in the group. The spatial evolutionary game theory has been developed by adding the spatial dimension to the classical evolutionary game [2, 3, 4]. Individual players arranged in the spatial lattice are characterized by the strategies which they take. Every individual plays games with his neighbors, and they reconstruct their strategies following some rules related to the payoff of their games. The system evolves by carrying out the above reconstruction simultaneously and iteratively. Therefore, the spatial evolutionary game is considered as a cell automaton, which recently attracts attention as an important model dealing with the discrete dynamic system in complex system physics[5].

One of the simplest and interesting evolutionary games is the prisoner's dilemma game, which is a classical non-zero-sum two-person game. Each individual chooses either a cooperative strategy or that of defection in their games. They can get the following payoffs corresponding to their strategies. A cooperator gets payoff Q by playing with a cooperator, and S by playing with a defector. In contrast, a defector gets payoff P by playing with a cooperator, and R by playing with a defector. If $P > Q > R > S$, both always defect as a consequence of logic, in spite that they can benefit more by cooperating each other. This is the dilemma.

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In the spatial evolutionary prisoner's dilemma game, it was shown that cooperators and defectors could coexist, whereas they were incompatible in a homogeneous system. Furthermore, it was reported that this model presented many kinds of spatial and temporal patterns [2]. One of the characteristics of a deterministic cell automaton like this is that the behaviors of a system depend on the details of local spatial structure. Thus, the mean field analysis cannot be applicable, which is used in a homogeneous evolutionary game theory.

A study of dynamics of spatial evolutionary game from a standpoint of non-equilibrium statistical physics has been reported recently [6, 7, 8]. Killingback and Doebeli pointed out that self-organized critical state exists in a spatial Hawk-Dove game. So far, it was known that the spatial prisoner's dilemma game had only periodic or chaotic dynamics, and did not show a critical state [2, 3]. However, Szabo and Toke clarified that the spatial prisoner's dilemma game exhibits a continuous transition by introducing a stochastic choice of strategies [7].

This paper is to investigate what happens in the spatial evolutionary prisoner's dilemma game by introducing random traveling of individuals. One of the reasons to introduce the concept of diffusion is that it is quite natural for most creatures in the actual ecosystem. The second one is to extract the characteristics of the dynamics by introducing a stochastic element to the system.

Diffusive traveling of players contributes to homogenization of the system. The effects that this homogenization causes to the coexistence of cooperators and defectors will be investigated. The traveling of individuals introduces a random fluctuation to the system with complex local interaction between the constituents. From a viewpoint of statistical physics, it is an interesting problem what kinds of dynamical characteristics are induced by giving the random force to the nonlinear system. Also, it seems interesting to identify what kind of results will be brought by the external compulsion of cooperation in spatial evolutionary games [8]. Therefore, the effect of external force to self-organization will be discussed.

In the next section, a spatial evolutionary prisoner's dilemma game without traveling of individuals is described. The problems of coexistence of cooperators and defectors in a system with random traveling of individuals are considered in section 3. In section 4, the dynamic characteristics of the coexistent state of the traveling PDG are discussed. The effect of the external forces on the system is considered in section 5. Section 6 is devoted to the conclusion.

2 Spatial Evolutionary Prisoner's Dilemma Game

An ordinary spatial evolutionary prisoner's dilemma game without a diffusion of players (hereafter 'fixed PDG') is to be considered in this section[2, 3, 4]. Players are placed on each site of a $n \times n$ square lattice, and adopt either a cooperative strategy or that of defection. As an initial state, cooperators are located at random at the probability q, and defectors are posted at the rest of the sites.

The state $s_i(t_j)$ of site i at discrete time $t_j, (j = 0, 1, 2, \cdots)$ is specified by the strategy of the player who occupies that site. The state $S(t_j)$ of the system at a time t_j is decided by specifying the states of all the sites. If the time evolution rule which decides $S(t_{j+1})$ for $S(t_j)$ is settled, the dynamical process is defined by giving the initial state $S(t_0)$.

The rule of time evolution is defined as follows. Each player plays a prisoner's dilemma game respectively with their second nearest neighbors on eight sites (i.e. Moore neighborhood sites). The periodic boundary condition is assumed to remove an influence at the end of lattice. The players get the payoff in accordance with the following payoff matrix of the game. In other words, a cooperator gets 1 against a cooperator, but nothing against a defector. A defector get $p (p > 1)$ against a cooperator, but nothing against a defector. The advantage for defectors, p, is the characteristic parameter of the time evolution of the system. Each player's total payoff is given by amount of the payoff in each play on eight Moore neighborhood sites. The state $s_i(t_{j+1})$ of the site i at time t_{j+1} is renewed by the strategy of the player who got the highest score at the Moore neighborhood sites including itself at time t_j.

	cooperate	defect
cooperate	(1, 1)	(0, p)
defect	(p, 0)	(0, 0)

Thus, the fixed PDG is two-state cellular automaton with a parameter p. The time evolution from t_0 to t_N $(= N\Delta t)$ by the time interval Δt gives the discrete dynamic process $(S(t_0), S(t_1), \cdots, S(t_N))$. The numbers of sites occupied by cooperators and defectors at time t_j are denoted respectively as $c(t_j)$ and $d(t_j)$, where $c(t_j) + d(t_j) = n^2$. The proportion of cooperators $x(t_j) = c(t_j)/n^2$ at discrete time t_j is used as a dynamical variable.

The behavior of time series $x(t_j)$ obtained by evolving the system with lattice size $n = 64$ depends on the parameter p. The initial configuration is constructed by giving the probability q. Figure 1 shows two typical behaviors of time series $x(t_j)$ which start from the same initial arrangement

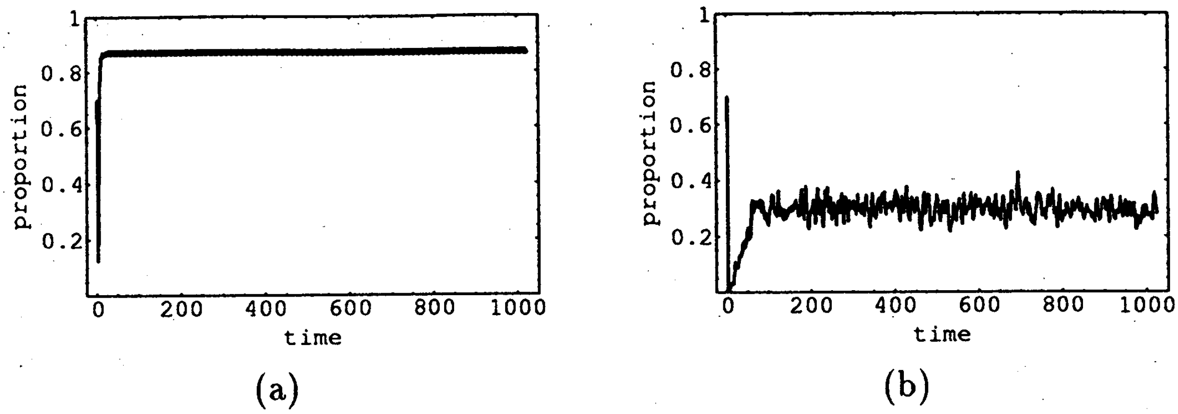

Figure 1: The time series of the proportion of cooperators for $p = 1.23$ (a) and $p = 1.63$ (b) in the fixed PDG.(n=64,q=0.7,N=1024)

with $q = 0.7$. It is clear that the system settles down in a periodic state for $p = 1.23$, and a chaotic state for $p = 1.63$. Coexistence of cooperators and defectors is possible due to the spatial structure, which is not possible in the case of homogeneous systems.

From the lattice configuration of steady state at various values of p and q, we can observe the influence of the initial arrangement (which depends to the value of the probability q) to the steady state and the bifurcation of steady state by the parameter p. In Figure 2, we denote cooperator's site by black, and defector's site by gray. When the initial proportion is small($q < 0.4$), a delicate difference of the initial arrangement influences the steady state. However, when q is not so small, it is assumed that essential characteristics of the system appears independently of its initial arrangement. If $p < 1.6$, the system shows a periodic state, whereas a chaotic state appears for $1.6 \leq p < 1.7$. If $1.7 \leq p$, cooperators do not appear. In particular, the system forms spontaneously a stable spatial structure in the periodic state.

In order to investigate the dependence of the behavior of the system to the parameter p more precisely, the averaged proportion of cooperators x against p is shown for the fixed PDG with the same initial configuration. In Figure 3 there are many discrete transitions within the coexistent states($0 < x < 1$).The average values x vary a little according to the initial configuration, but a transition point does not vary.

In consequence of the spatial arrangement of their neighborhood, the total payoff of a cooperator is either $1, 2, \cdots,$ or 8, and that of a detector is either $p, 2p, \cdots,$ or $8p$. This discrete property of a possible total payoff results in the discrete transition points for parameter p. There are ten threshold values for p, $\{8/7, 7/6, 6/5, 5/4, 4/3, 7/5, 3/2, 8/5, 5/3, 7/4\}$, in the region of $1 < p < 2$. In the region of $8/7 \leq p \leq 3/2$, transitions

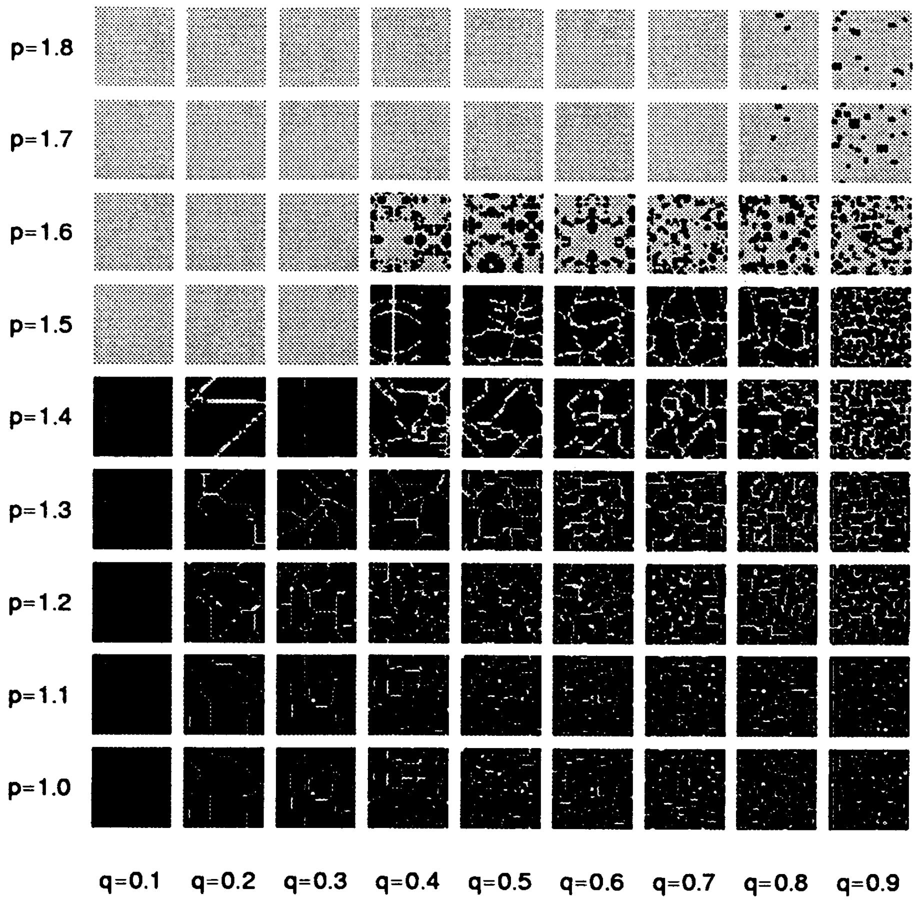

Figure 2: The steady state configurations in the fixed PDG. Black and gray boxes denote cooperators and defectors respectively. ($n = 64$)

occur during the periodic states. The system changes to the chaotic state at $p = 8/5$, to the quiescent state at $p = 5/3$, and to the extinct state at $p = 7/4$.

In the spatial evolutionary game theory, the probability that a cooperator plays with a defector is not given by the product of their proportions to total population, but depends on the details of surrounding spatial structures. As a result of this fact, the coexistence of cooperators and defectors comes to be possible in contrast to the homogeneous evolutionary game theory. The interaction reflecting a local spatial structure becomes essential in the spatial evolutionary game, therefore, the mean field analysis turns to be useless.

We can indicate the following behavior of small clusters in the present system. One defector surrounded by cooperators oscillates with period 2 (1D $\rightarrow$ 9D $\rightarrow$ 1D) for $1 < p < 6/5$, and oscillates with period 3 (1D $\rightarrow$ 9D

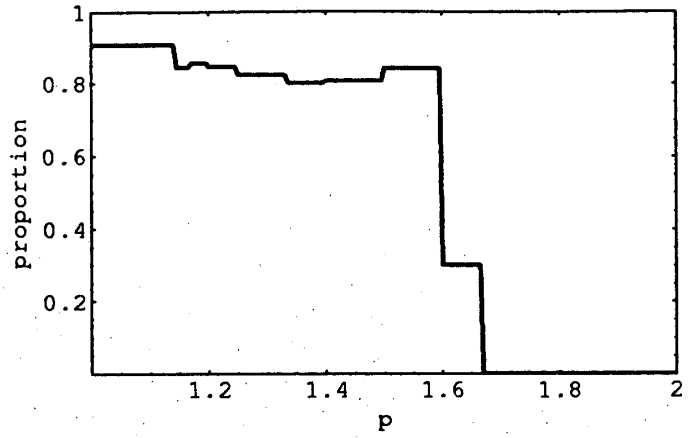

Figure 3: The averaged proportion of cooperators as a function of p in the fixed PDG.(n=64,q=0.7,N=128)

$\rightarrow$ 5D $\rightarrow$ 1D) for $6/5 < p < 7/5$. It reposes (9D) for $7/5 < p < 8/5$ without growing nor disappearing, and grows for $8/5 < p < 2$, where 9D and 5D denote a 3×3 cluster and a cross of defectors respectively. A 2×2 cluster of 4 defectors (4D) in the sea of cooperators disappears for $1 < p < 7/5$, reposes for $7/5 < p < 8/5$, oscillates with period 3 for $8/5 < p < 5/3$, and grows for $5/3 < p$. A cross of 5 defectors (5D) oscillates with period 2 (5D $\rightarrow$ 1D $\rightarrow$ 9D $\rightarrow$ 1D) for $1 < p < 6/5$, oscillates with period 3 (5D $\rightarrow$ 1D $\rightarrow$ 9D $\rightarrow$ 5D) for $6/5 < p < 7/5$, reposes for $7/5 < p < 8/5$, oscillates with period 7 for $8/5 < p < 5/3$, reposes for $5/3 < p < 7/4$ also for $7/4 < p < 2$. 9D oscillates with period 2 (9D $\rightarrow$ 1D $\rightarrow$ 9D) for $1 < p < 6/5$, oscillates with period 3 (9D $\rightarrow$ 5D $\rightarrow$ 1D $\rightarrow$ 9D) for $6/5 < p < 7/5$, reposes for $7/5 < p < 8/5$, and grows for $8/5 < p < 2$.

On the one hand, a cluster of less than three cooperators surrounded by defectors disappears always for $p > 1$. A 2×2 cluster of 4 cooperators (4C) grows for $1 < p < 3/2$, and disappears for $p > 3/2$. A 3×3 cluster of 9 cooperators (9C) grows for $1 < p < 3/2$, reposes for $3/2 < p < 8/5$, disappears for $8/5 < p < 5/3$, and reposes for $p > 5/3$.

From the above analysis, it is clear that there exist some threshold values of parameter p related to the growth and extinction of various small clusters. We consider that these threshold parameters, especially $p_{c1} = 1.5$ of 4C and 9C, $p_{c2} = 1.6$ of 1D and 9D, and $p_{c3} = 1.67$ of 4D, play an important role to determine the steady state reached by the system.

3 Effects of the Diffusion of Individuals

In the fixed PDG, a player located on a lattice site changes its strategy as a result of game with other players in the neighborhood sites. This is a suitable model for a group, for instance, the plant ecosystem, where their members are spatially fixed and interact with only their neighbors. But, the movement of members cannot be ignored in animal ecosystems, human society, and others. Therefore, we will introduce diffusive traveling of individuals into the fixed PDG. This is called the 'traveling PDG' hereafter.

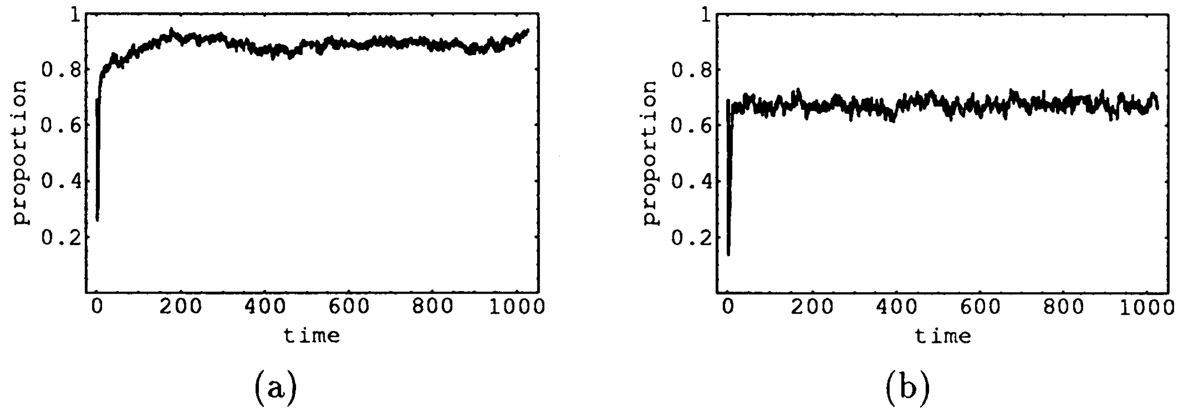

Figure 4: The time series of the proportion of cooperators for $p = 1.16$ (a) and $p = 1.23$ (b) in the traveling PDG.(n=64,q=0.7,N=1024)

A player located at the site of a two-dimensional square lattice takes either a strategy of cooperation or defection, and at the same time each player turns to either of four nearest-neighbor sites. Therefore, the state $w_i(t_j)$ of the site i at time t_j is specified by two variables, $w_i(t_j) = \{s_i(t_j), d_i(t_j)\}$, where $s_i(t_j) = \{C, D\}$ and $d_i(t_j) = \{N, E, S, W\}$ denote the strategy and direction, respectively.

The time evolution of the system is done by the following two steps. First, the renewal of strategy $s_i(t_j)$ according to the total payoff by the game is performed quite similarly with the fixed PDG described in the previous section. Secondly, according to the direction $d_i(t_j)$, each player exchanges the site each other only when they stand opposite to each other in the nearest neighbor sites. Therefore, in this case, the strategy of their sites is replaced with each other. Both steps are carried out simultaneously. After that each player takes a new direction set up at random. In other words, the traveling of player is done completely at random, independently to its strategy. The system evolves by repeating these rounds.

The proportion $x(t_j)$ of cooperators to the whole population at time t_j is a discrete dynamical variable, similarly to the fixed PDG. Figure 4 shows the time series $x(t_j)$ for $p = 1.16$ and $p = 1.23$. The proportion fluctuates continuously around the average.

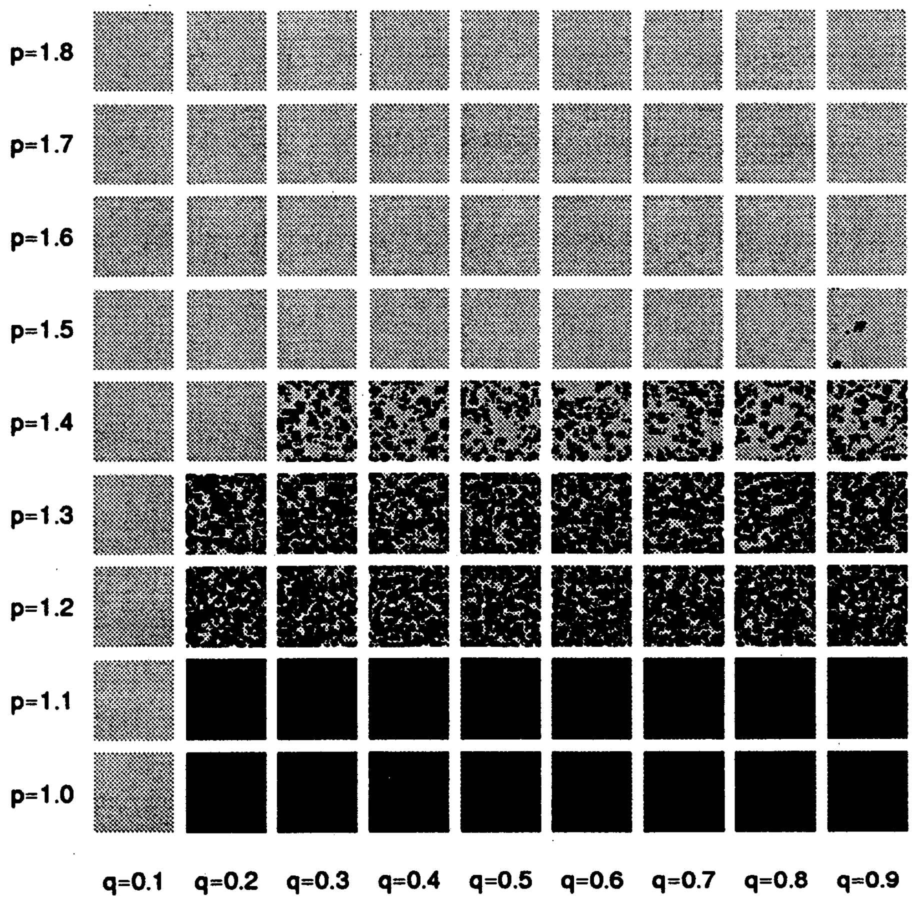

Figure 5: The steady state configurations in the traveling PDG. Black and gray boxes denote cooperators and defectors respectively. $(n = 64)$

Depending on the values of the probability q and the parameter p, the lattice configuration formed fully after a long time is shown in Figure 5. Comparing with Figure 2 in the case of the fixed PDG, it is understood that the diffusion of individuals makes it difficult for cooperators to survive, but not impossible. The difference of the initial configuration has no important effects except in the case of remarkably small q. We set $q = 0.7$ in the following analysis.

In Figure 6, the rate of the population of cooperators is shown as a function of the parameter p. The homogeneous state ($p \leq 1.14$), the annihilation state ($1.50 < p$) of cooperators and the coexistent states ($1.14 < p < 1.50$) are observed. Discontinuous transitions among six coexistent states occur at $p = 1.167$, $p = 1.20$, $p = 1.25$, $p = 1.33$, $p = 1.40$.

Each coexistent state is steady and the values of the proportion of cooperators are $x \sim 0.88,\ 0.76,\ 0.67,\ 0.65,\ 0.44,\ 0.34$, respectively.

Investigating the time series repeating alternately the fixed and traveling PDG at regular intervals verified the stability of the coexistent state. For a given parameter p, the fluctuating proportion of cooperators in the coexistent state of the traveling PDG has an almost constant averaged value. In the traveling PDG, the steady state is independent of the initial state, and there exists the homogeneous state of cooperators in the region $1 < p < 1.14$. These results coincide with the result by Szabo and Toke[7]. They have introduced a stochastic factor to their model. It is interesting that the proportion of cooperators is larger for the traveling PDG than for the fixed PDG in the region $p < 1.167$. The cooperators in the traveling PDG are impossible to exist in $p > 3/2$. This threshold value $p = 3/2$ is the point where the square clusters 4C and 9C cannot grow. It is likely that the diffusive traveling of individuals cuts and deforms the self-organized structure, and makes the existence of cooperator clusters difficult.

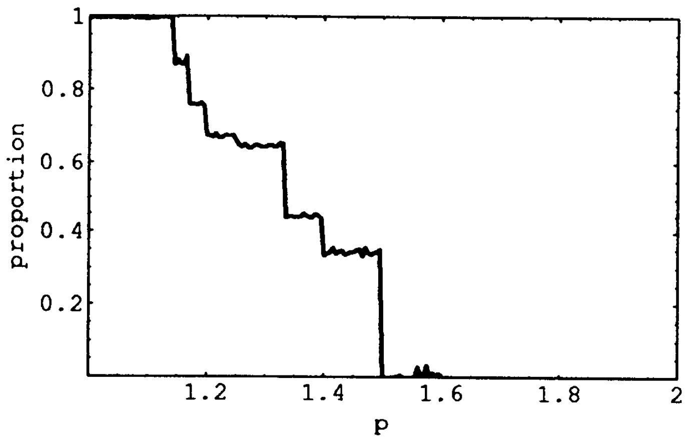

Figure 6: The averaged proportion of cooperators as a function of p in the traveling PDG.(n=64,q=0.7,N=128)

4 Dynamic Characteristics of the Coexistent State

In order to study the dynamic characteristics of a system, the power spectrum of time series is useful. The discrete Fourier transform of time series $x(t_j),(j=1,2,\cdots,N)$ is defined by

$$X(f_k)=\frac{1}{\sqrt{N}}\sum_{j=1}^{N}x(t_j)e^{i2\pi(j-1)(k-1)/N},\quad (k=1,2,\cdots,N),$$

where f_k is a discrete frequency, and $f_0=0$. The power spectrum of time series $x(t_j)$ is given by

$$P(f_k)=|X(f_k)|^2.$$

We can say the following features on the power spectrum. (i) If the time series is periodic or quasi-periodic with period T, the spectrum has a sharp peak at frequency $1/T$. (ii) In the case of non-periodic time series, there is a frequency region where the spectrum decreases by a power of frequency

$$P(f_k)\sim f_k^{-\gamma}.$$

The power γ shows the strength of time correlation. (iii) In the region of frequency where the spectrum is independent of frequency, the time series has no correlation and is called white noise. The white noise spectrum appears generally in a low frequency, and the correlation time, which is a maximum time remaining time correlation, gives the time constant characterizing the physical origin of fluctuation.

The power spectrum of time series in the fixed PDG is plotted on a log-log graph in Figure 7. The spectrum for $p=1.23$ has peaks corresponding to periods 3,4 and 6. In the case of $p=1.63$, the power spectrum does not depend on the frequency in a wide region, which means that the state is chaotic.

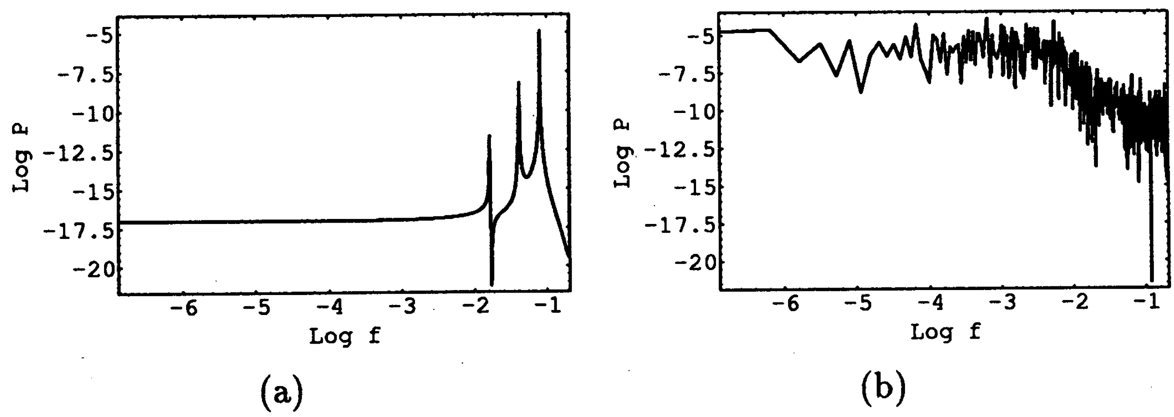

Figure 7: The power spectrum of $x(t_j)$ for $p=1.23$ (a) and $p=1.63$ (b) in the fixed PDG.(n=64,q=0.7,N=1024)

In the traveling PDG, the average of the power spectrums for 20 sets of time series with the same initial configuration is considered. Figure 8 is the averaged power spectrum for $p = 1.16$,$p = 1.18$,$p = 1.23$,$p = 1.29$,$p = 1.37$,$p = 1.45$, which belong to six coexistent states respectively. From the feature of the power spectrum, the coexistent states are divided into two regions as follows: (I) the parameter range ($8/7 < p < 7/6$), where the spectrum obeys a power-law. (II) the parameter range ($7/6 < p < 3/2$), where the spectrum obeys Lorentzian-form.

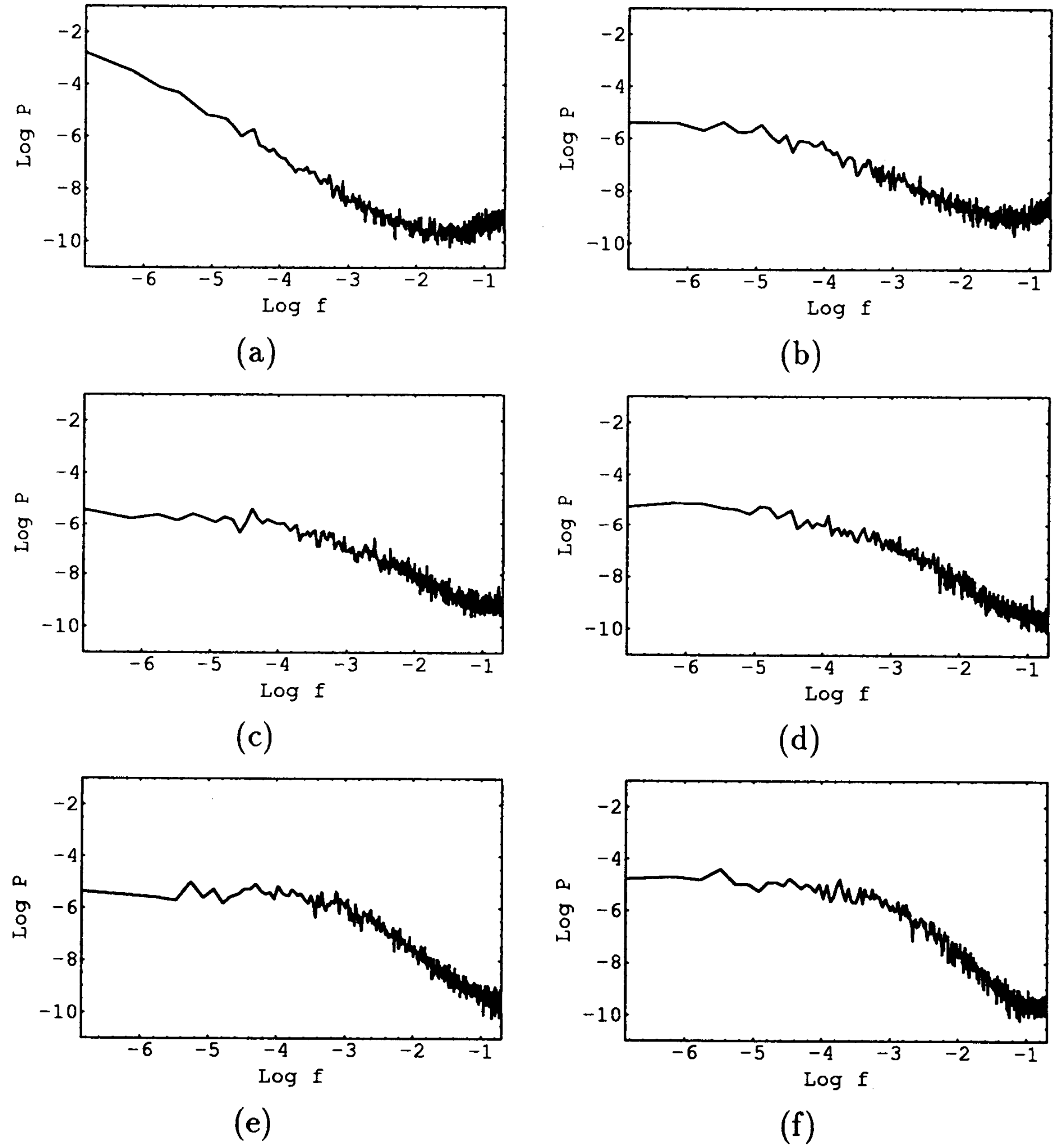

Figure 8: The power spectrum of $x(t_j)$ for $p = 1.16$(a), $p = 1.18$(b), $p = 1.23$(c), $p = 1.29$(d), $p = 1.37$(e) and $p = 1.45$(f) in the traveling PDG.

The power exponent γ in the region (I) is that $\gamma \sim 1.4$. It means that the correlation of every time scale exists in the time series $x(t_j)$, which is characteristics for the critical state. In the region (II) of Lorentzian spectrum, $\gamma \sim 1.1$ (for $7/6 < p < 6/5$), $\gamma \sim 1.3$ (for $6/5 < p < 5/4$), $\gamma \sim 1.5$ (for $5/4 < p < 4/3$), $\gamma \sim 1.9$ (for $4/3 < p < 7/5$), $\gamma \sim 2.1$ (for $7/5 < p < 3/2$). The time correlation increases with the value of p. This means that the fluctuation introduced by random traveling of individuals is turned to the fluctuation with strong correlation by local interaction of the system.

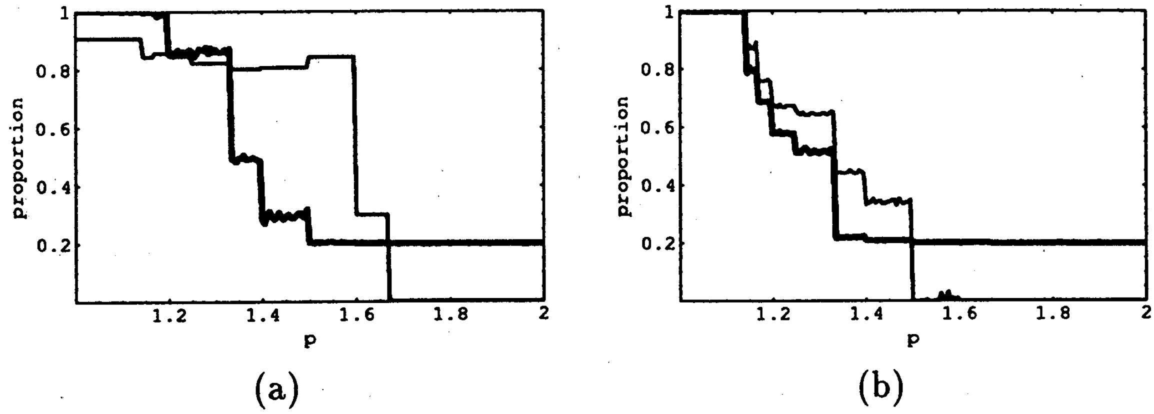

Figure 9: The proportion of cooperators as a function of p, in the fixed PDG (a) and in the traveling PDG (b). Thin and thick lines show the non-forced case and the case with external compulsion respectively. ($u = 0.2$)

5 Effects of External Compulsion

The influence of the external force to cooperate is interesting as a practical problem. We assume that individuals are forced to cooperate stochastically at each round of a spatial prisoner's dilemma game. To put it concretely, the strategy of an individual selected at random in a probability u is changed to cooperate, if it is defect; but it is not changed, if it is cooperate. This step in the time evolution is called the external compulsion, and u is a parameter which denotes the degree of compulsion.

The averaged proportion of cooperators in the steady state except for the transient region is represented against parameter p in figure 9(a) and 9(b), where the figure 3 and 6 without external forces are shown together.

In figure 9(a) of the fixed PDG, the external compulsion to cooperate brings negative effects except in the region of small p. At $p > 1.5$, the proportion falls into the level only of external probability. This fact means that the spatial local structure is important for cooperators to survive, and the random force to cooperate brings the opposite effect.

It will be noticed that in the traveling PDG the random compulsion to cooperate brings negative effects in the region of all p. This suggests that the random external compulsion is fatal rather than the diffusive traveling of individuals to break the spatial local structure. It will be didactic that forcing indiscriminately individuals to cooperate does not produce any results that are expected, but a contrary effect obstructing the spontaneous emergence of cooperation.

6 Conclusions

The effects of the random traveling of individuals in the spatial evolutionary prisoner's dilemma game were investigated as contrasted with the game without traveling. The annihilation state where cooperators are wiped out, arose at smaller values of p ($1.5 \leq p$) for the traveling PDG than that ($1.67 \leq p$) for the fixed PDG. The fixed PDG produced the periodic state at $1.5 \leq p < 1.6$, and the chaotic state at $1.6 \leq p < 1.67$. If random traveling is introduced to this system, the circumstances around each individual will become to be similar to that of a homogeneous system from the point of view of the mean field approximation. Thus, it is probable that the annihilation state arises in this region of the traveling PDG.

In the traveling PDG, the coexistent state appeared in the region of $1.14 < p < 1.5$, between the homogeneous state and the annihilation state. The coexistent state in the fixed PDG was the periodic state for $1 \leq p < 1.6$ and chaotic state for $1.6 \leq p \leq 1.67$. It was pointed out that the parameter range of the coexistent state in the traveling PDG was divided into two ranges which were (I) a power-law spectrum range ($8/7 < p < 7/6$) and (II) Lorentzian spectrum range ($7/6 < p < 3/2$). In the region (I), the power exponent γ was equal to 1.4. The power-law spectrum means that the long time correlation exists in the time series $x(t_j)$, which is characteristics for the critical state. The range (I) was a region where the proportion of cooperators was larger in the traveling PDG than in the fixed PDG. From these results, it is suggested that random traveling of individuals in this parameter range leads to the effect organizing the cluster of cooperators.

The effects of external compulsion are paradoxical. It has been clarified that the random external force does not bring any positive results in the self-organized system by the local interaction.

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